

EE 330

Lecture 16

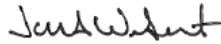
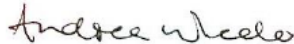
Devices in Semiconductor Processes

- Diodes (continued)
- Capacitors
- MOSFETs

Interoffice Communication

Date: September 29, 2021

To: Iowa State University Faculty

From: Jonathan Wickert
Senior Vice President and Provost 
Andrea Wheeler
President, Faculty Senate 

Subject: Public Health Messaging for Students via Canvas

Iowa State University's public health team is launching a series of weekly messages to encourage students to get vaccinated and wear a mask as well as inform them about the spread of COVID-19 on campus and in the community.

Similar to the Cyclones Care campaign, the messages were developed with input from the Faculty Senate Executive Board, and in particular Meghan Gillette, who has expertise in communication with college-age adults. The messages will be posted on Canvas every Wednesday morning, following the weekly update to the [COVID-19: By the numbers](#) website, and will remain on the site for 24 hours. The messages will be displayed on the faculty dashboard in Canvas, so you can access them easily.

We encourage you to share this information with students during class or as part of routine messages and communications with students. The Canvas messages will include a link to the COVID-19: By the numbers website, if you would like to pull the weekly data to share.

We ask that department chairs share this memo with all instructors, including graduate student instructors, and teaching assistants, and academic advisors and staff engaged in instruction and student success.

Thank you for your work this semester and help in keeping our students informed.

C: Wendy Wintersteen, President
Pam Cain, Senior Vice President for Operations and Finance
Toyia Younger, Senior Vice President for Student Affairs
Provost's Council
Chris Johnsen, President, Professional and Scientific Council

<https://web.iastate.edu/safety/updates/covid19/numbers>

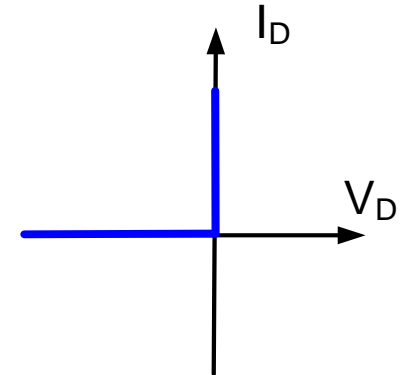
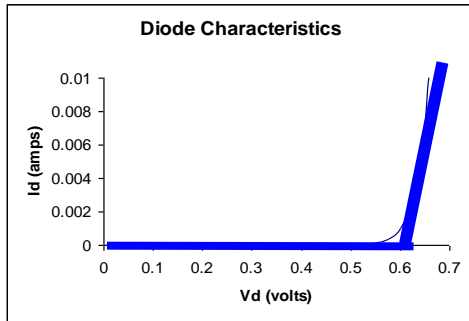
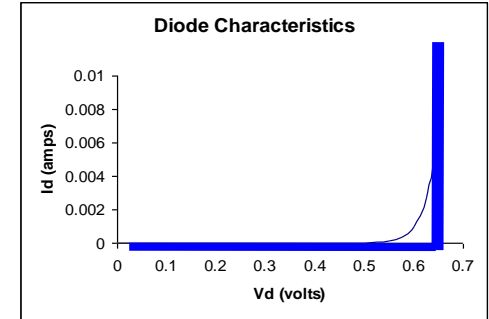
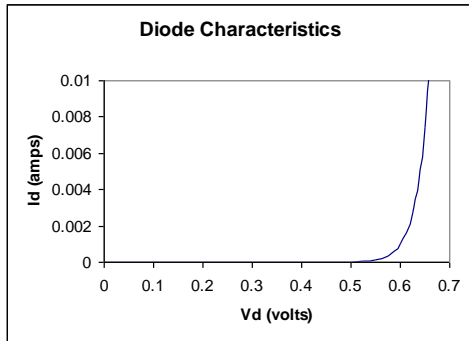
Photo courtesy of the director of the National Institute of Health (NIH)



As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

Diode Models



Which model should be used?

The simplest model that will give acceptable results in the analysis of a circuit

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

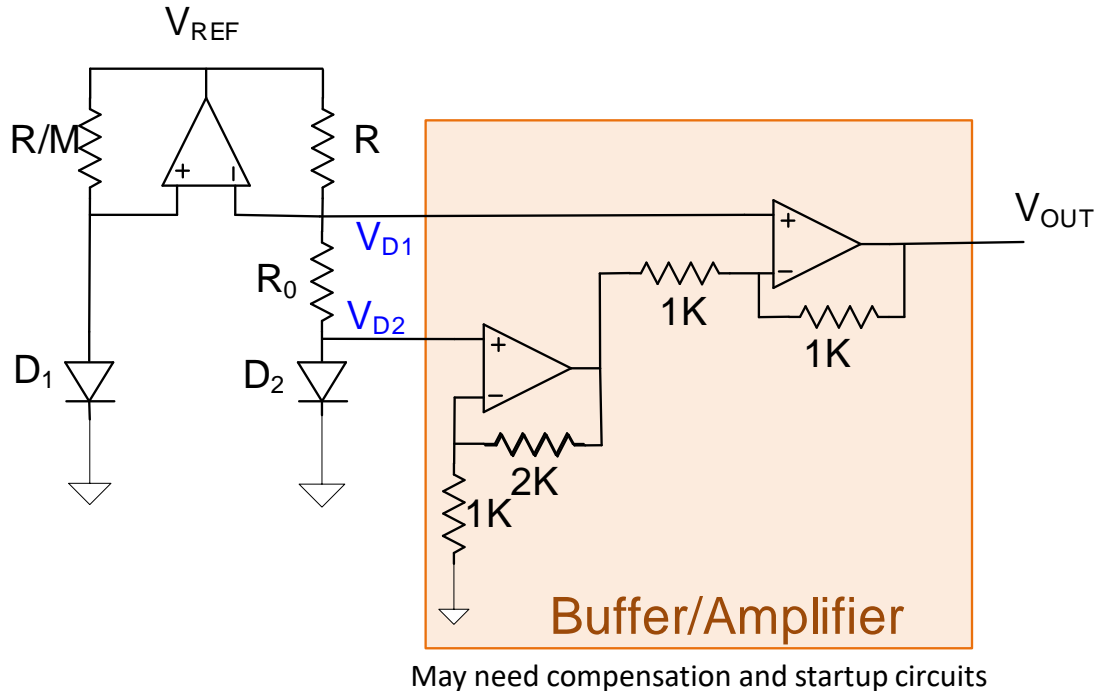
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Observations:

- Analysis generally simplified dramatically (particularly if piecewise model is linear)
- Approach applicable to wide variety of nonlinear devices
- Closed-form solutions give insight into performance of circuit
- Usually much faster than solving the nonlinear circuit directly
- Wrong guesses in the state of the device do not compromise solution (verification will fail)
- Helps to guess right the first time
- Detailed model is often not necessary with most nonlinear devices
- Particularly useful if piecewise model is PWL (but not necessary)
- For practical circuits, the simplified approach usually applies

Key Concept For Analyzing Circuits with Nonlinear Devices

A Diode Application



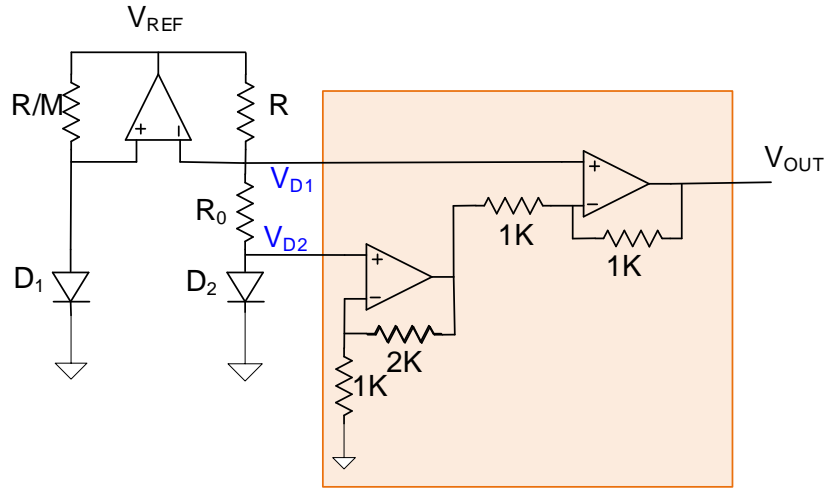
For appropriate R_0 , serves as bandgap voltage reference (buffer/amplifier excluded)

$$V_{REF} = V_{D1} + \frac{R}{R_0}(V_{D1} - V_{D2})$$

If buffer/amplifier added, serves as temperature sensor at V_{OUT}

$$V_{OUT} = 2(V_{D2} - V_{D1})$$

A Diode Application



$$V_{OUT} = 2(V_{D2} - V_{D1})$$

Analysis of temperature sensor (assume D_1 and D_2 matched)

$$\left. \begin{aligned} I_{D2}(T) &= \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D2}}{V_t}} \\ I_{D1}(T) &= \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D1}}{V_t}} \\ I_{D2}(T) &= M I_{D1}(T) \end{aligned} \right\}$$

$$V_t = \frac{k}{q} T$$

$$\rightarrow \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D2}}{V_t}} = M \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D1}}{V_t}}$$

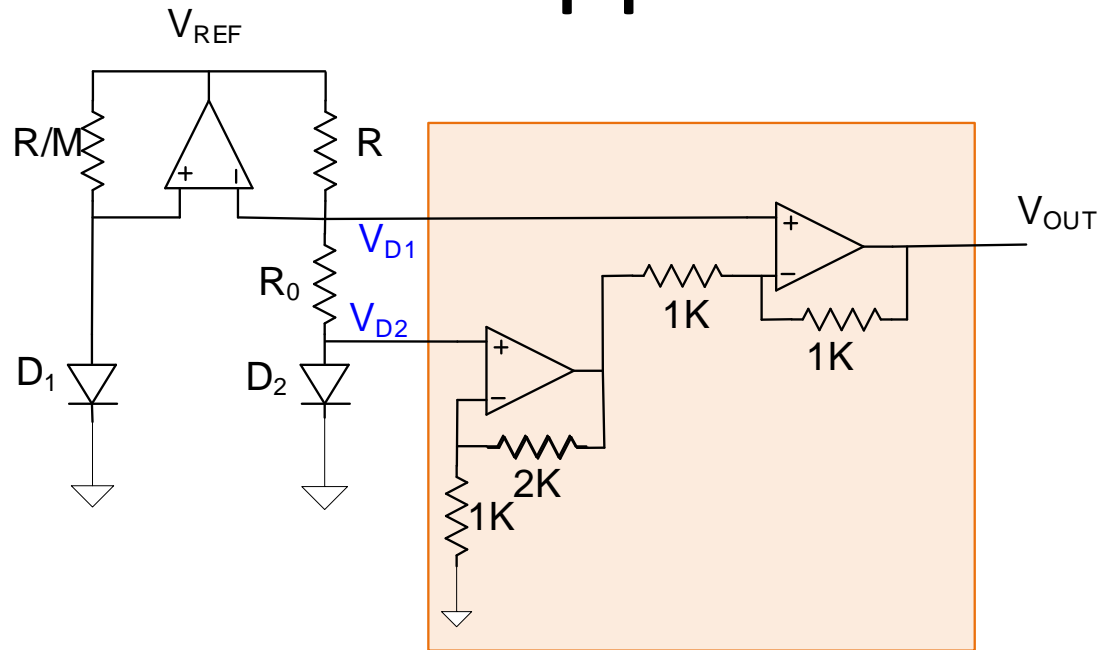
Cancelling terms and taking ln we obtain

$$V_{D2} - V_{D1} = V_t \ln M$$

Thus

$$V_{OUT} = 2(V_{D2} - V_{D1}) = 2 \ln M \cdot \frac{k}{q} T$$

A Diode Application



May need compensation and startup circuits

For appropriate R_0 , serves as bandgap voltage reference

$$V_{REF} = V_{D1} + \frac{R}{R_0}(V_{D1} - V_{D2})$$

If buffer/amplifier added, serves as temperature sensor at V_{OUT}

$$V_{OUT} = 2(V_{D2} - V_{D1})$$

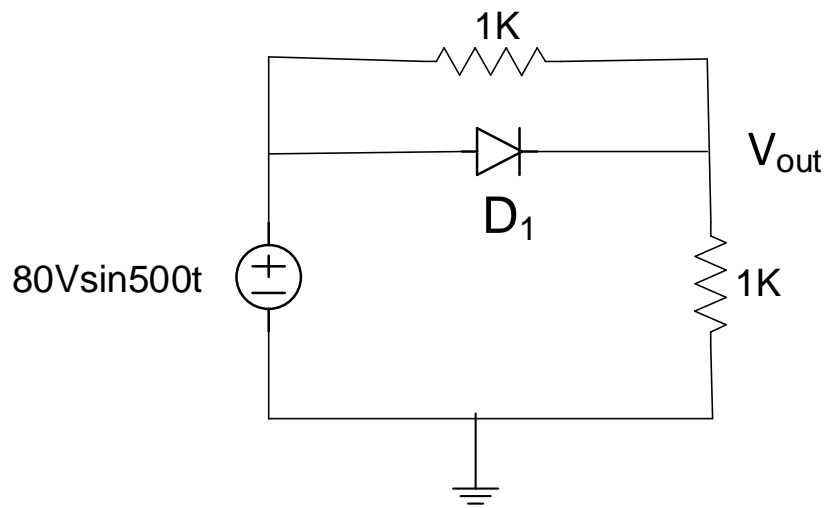
Analysis of V_{REF} to show output is nearly independent of T and V_{DD} is more tedious

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

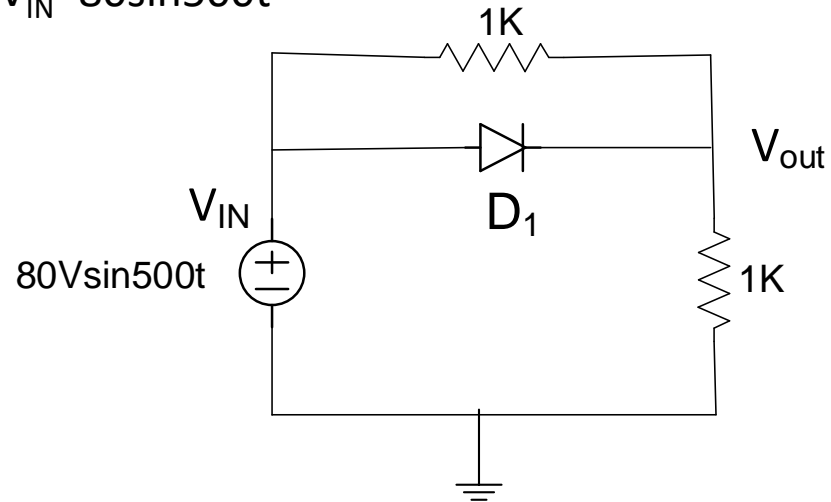
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about nonlinear circuits (using piecewise models) with time-varying inputs?

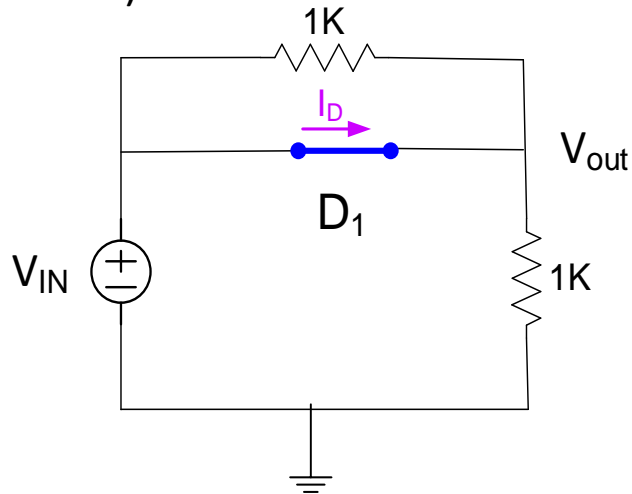


Same process except state verification (step 3) may include a range where solution is valid

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 ON (will use ideal diode model)

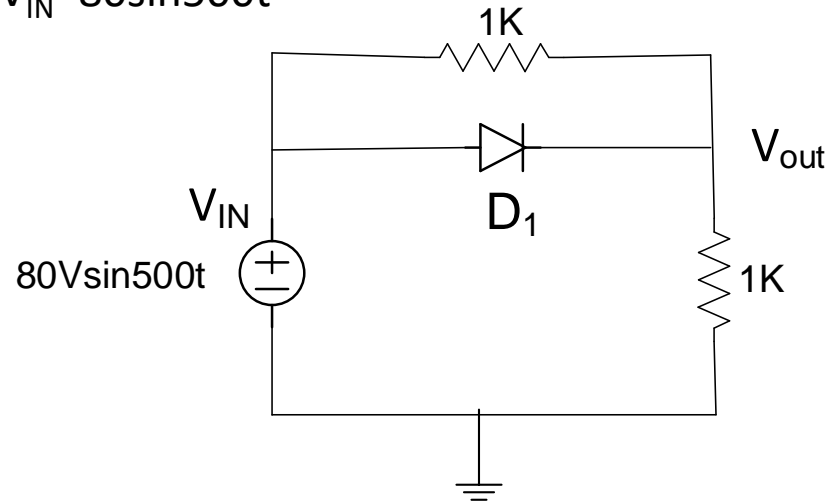


$$V_{OUT}=V_{IN}=80\sin(500t)$$

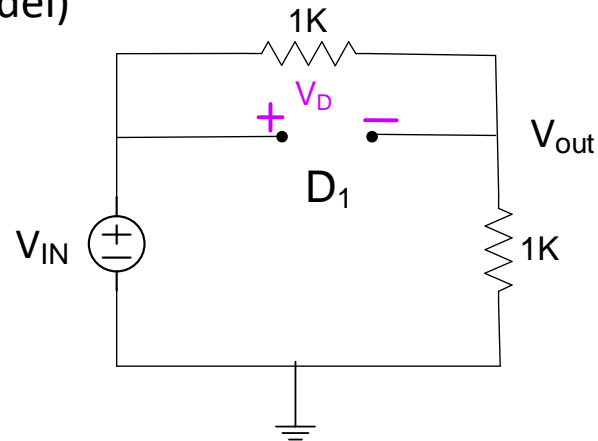
Valid for $I_D > 0$ $I_D = \frac{V_{IN}}{1K}$

Thus valid for $V_{IN} > 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 OFF (will use ideal diode model)

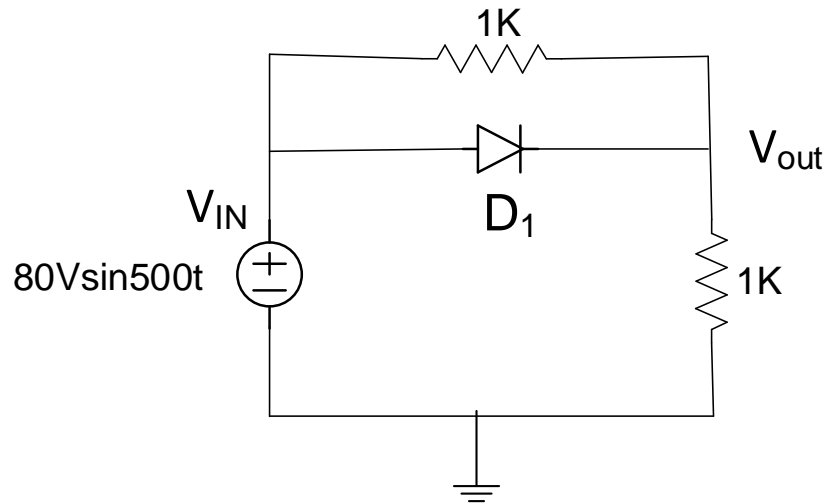


$$V_{OUT}=V_{IN}/2=40\sin(500t)$$

$$\text{Valid for } V_D < 0 \quad V_D = \frac{V_{IN}}{2}$$

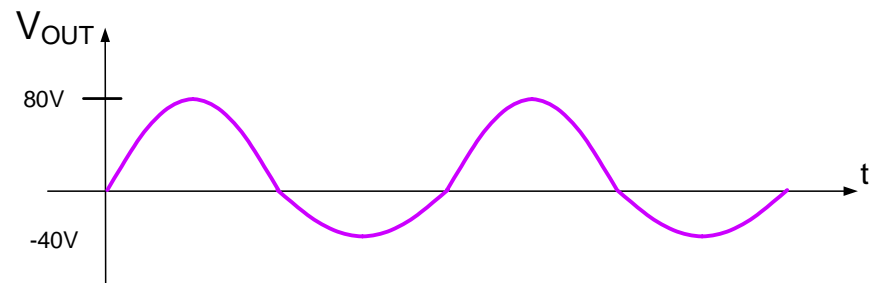
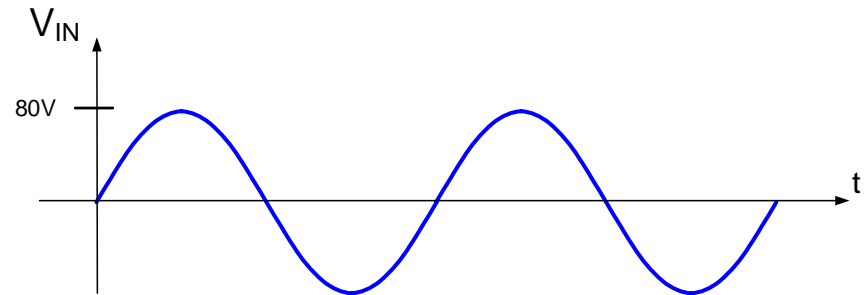
Thus valid for $V_{IN} < 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Thus overall solution

$$V_{OUT} = \begin{cases} 80 \sin 500t & \text{for } V_{IN} > 0 \\ 40 \sin 500t & \text{for } V_{IN} < 0 \end{cases}$$

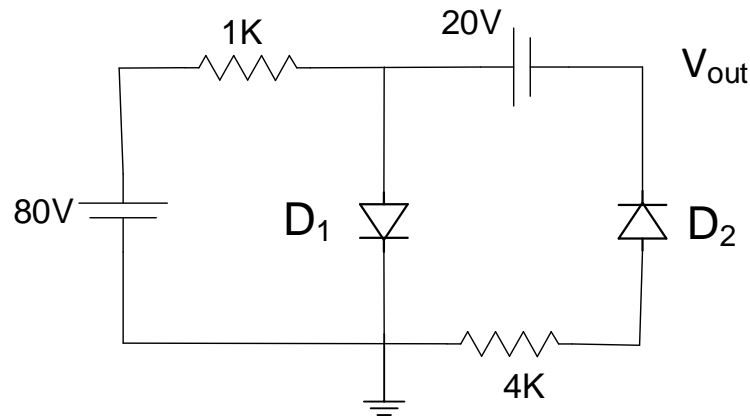


Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

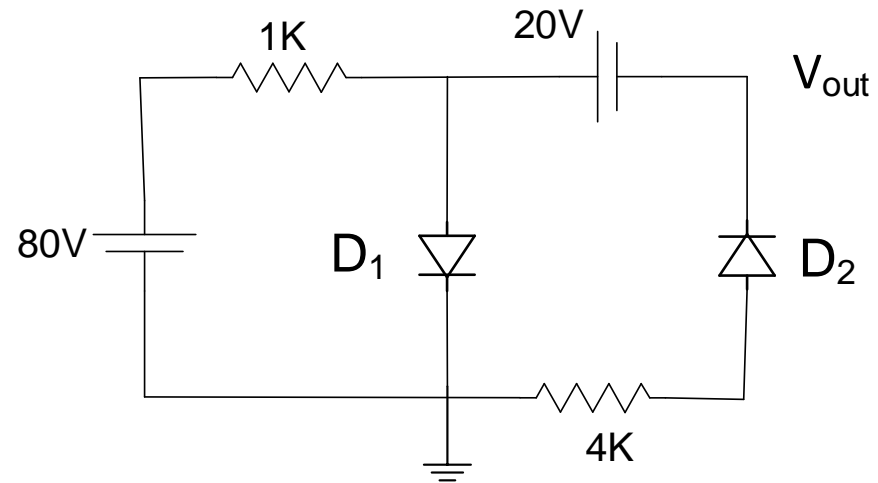
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about circuits (using piecewise models) with multiple nonlinear devices?

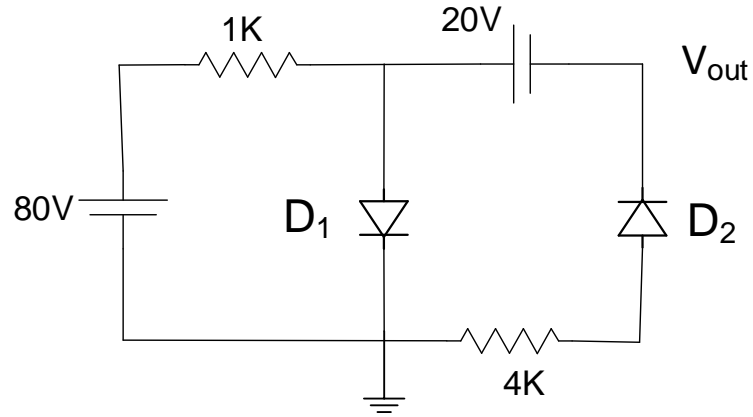


Guess state for each device (multiple combinations possible)

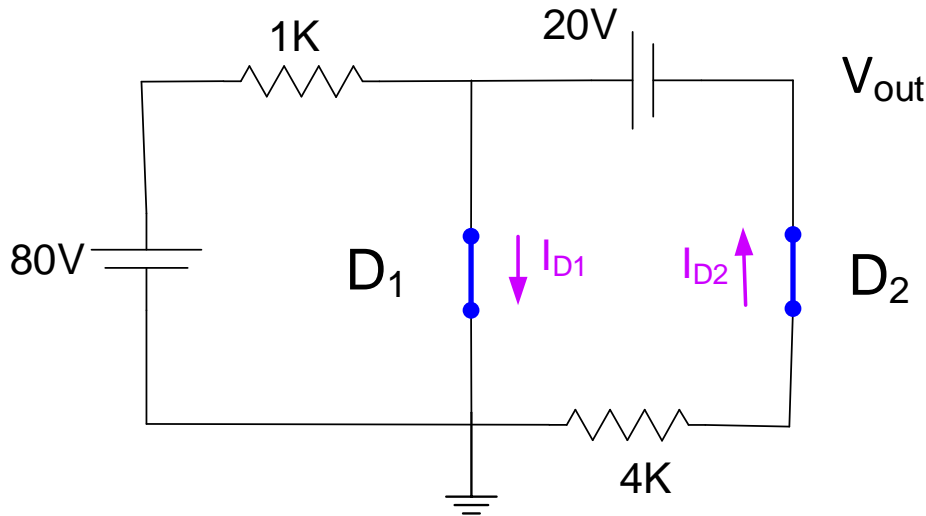
Example: Obtain V_{OUT}



Example: Obtain V_{OUT}



Guess D_1 and D_2 on



$$V_{OUT} = -20V$$

Valid for $I_{D2} > 0$ and $I_{D1} > 0$

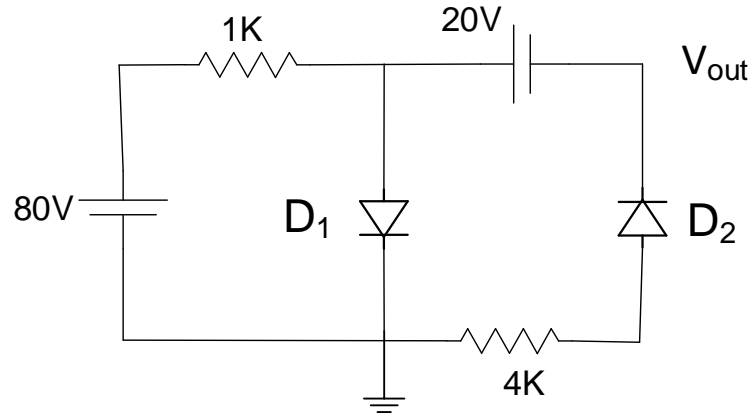
$$I_{D2} = \frac{20V}{4K} = 5mA > 0 \quad I_{D1} = \frac{80V}{1K} + I_{D2} = 85mA > 0$$

Validates

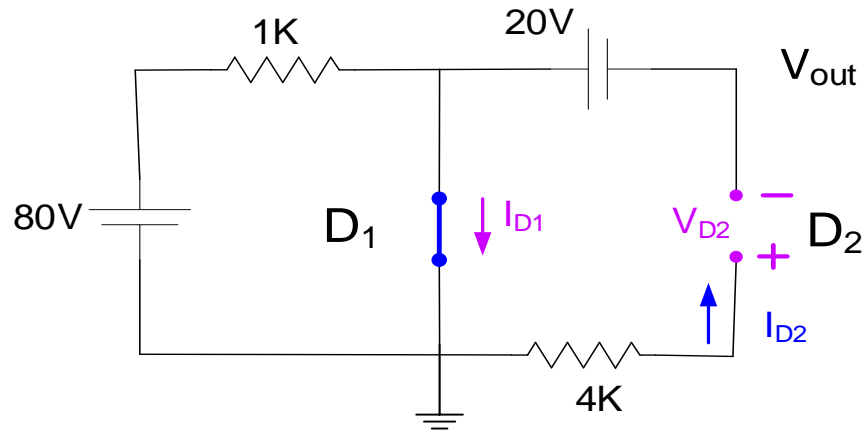
Validates

Since validates, solution is valid

Example: Obtain V_{OUT}



If we had guessed wrong
Guess D_1 ON and D_2 OFF



$$V_{OUT} = -20V$$

Valid for $I_{D1} > 0$ and

$$V_{D2} < 0$$

$$I_{D1} = \frac{80V}{1K} + I_{D2} = 80mA > 0$$

$$V_{D2} = +20$$

Validates

FAILS
Validation

Since fails to validate, solution is not valid so guess is wrong !

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Single Nonlinear Device

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

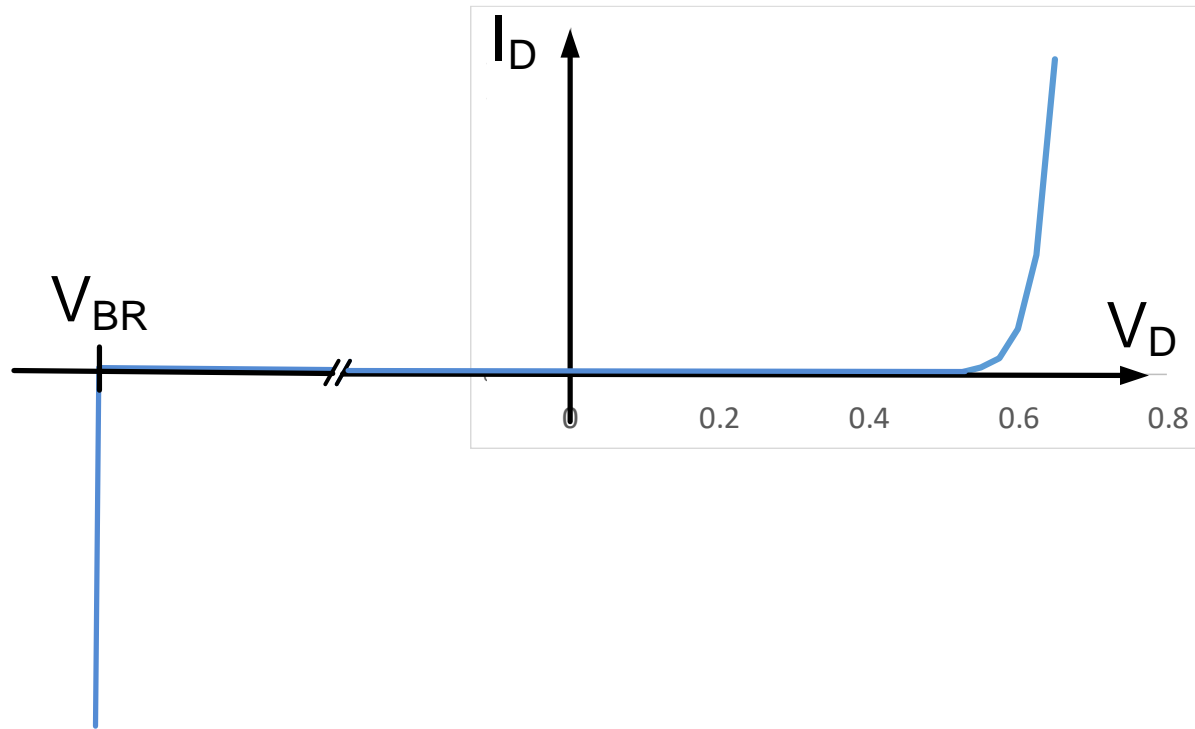
Multiple Nonlinear Devices

Process:

1. Guess state of each device (may be multiple combinations)
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify models (if necessary)

Analytical solutions of circuits with multiple nonlinear devices are often impossible to obtain if detailed non-piecewise nonlinear models are used

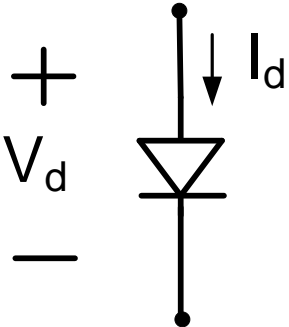
Diode Breakdown



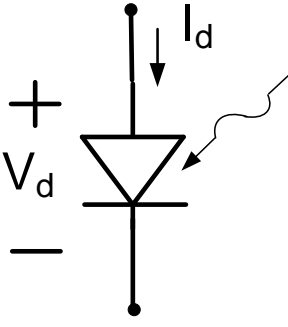
- Diodes will “break down” if a large reverse bias is applied
- Unless current is limited, reverse breakdown is destructive
- Breakdown is very sharp
- For many signal diodes, V_{BR} is in the -100V to -1000V range
- Relatively easy to design circuits so that with correct diodes, breakdown will not occur
- Zener diodes have a relatively small breakdown and current is intentionally limited to use this breakdown to build voltage references

Types of Diodes

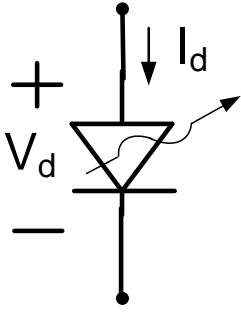
pn junction diodes



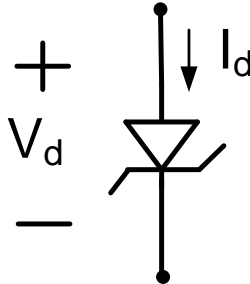
Signal or Rectifier



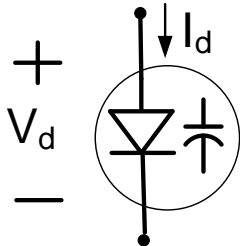
Pin or Photo



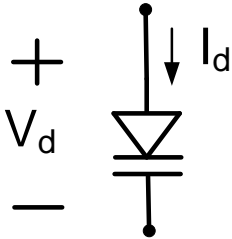
Light Emitting LED
Laser Diode



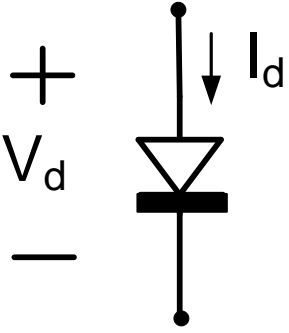
Zener



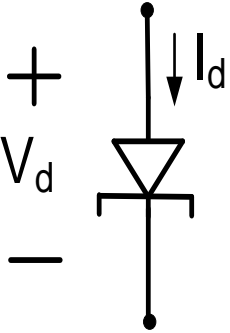
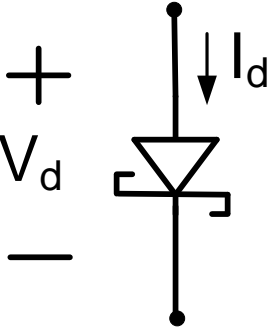
Varactor or Varicap



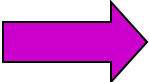
Metal-semiconductor junction diodes



Schottky Barrier



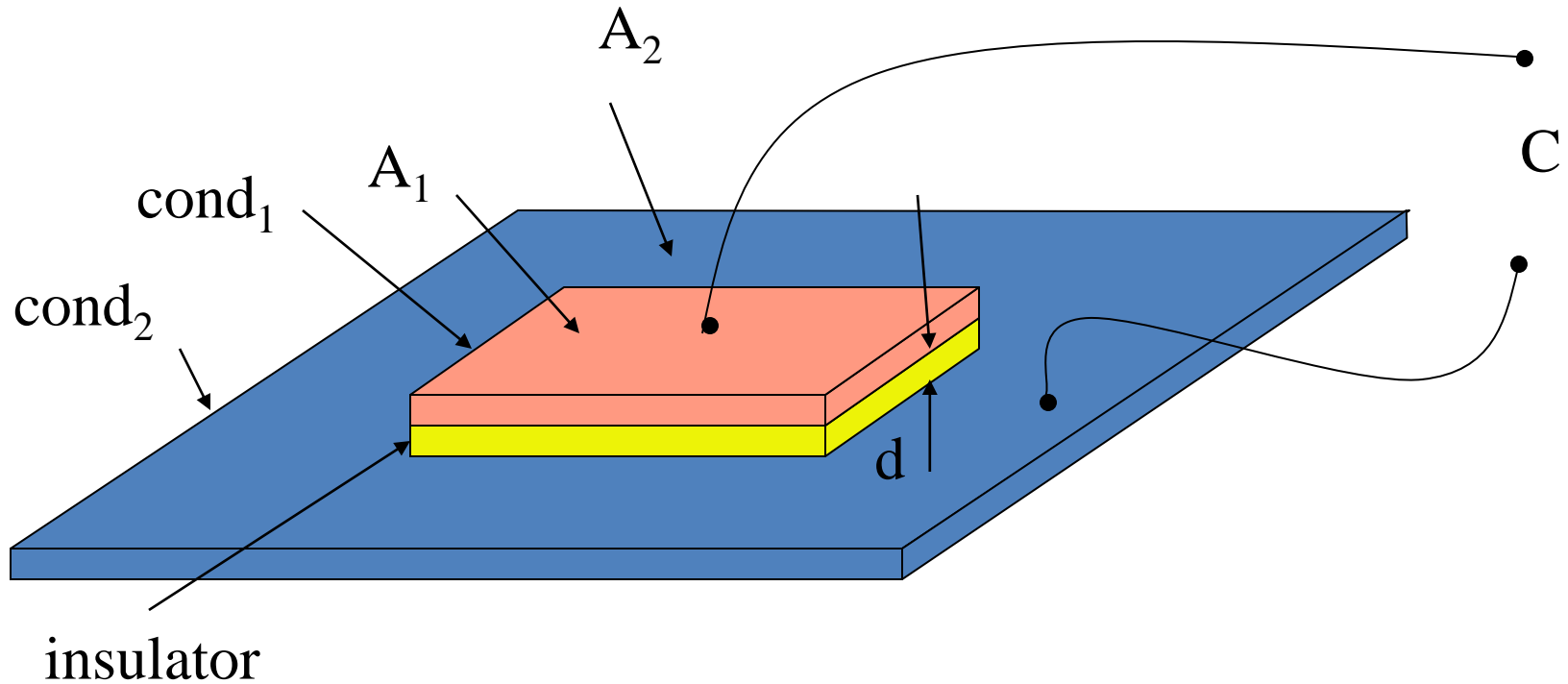
Basic Devices and Device Models

- Resistor
- Diode
-  Capacitor
- MOSFET
- BJT

Capacitors

- Types
 - Parallel Plate
 - Fringe
 - Junction

Parallel Plate Capacitors



A = area of intersection of A_1 & A_2

One (top) plate **intentionally** sized smaller to determine C

$$C = \frac{\epsilon A}{d}$$

Parallel Plate Capacitors

$$\text{If } C_d = \frac{\text{Cap}}{\text{unit area}}$$

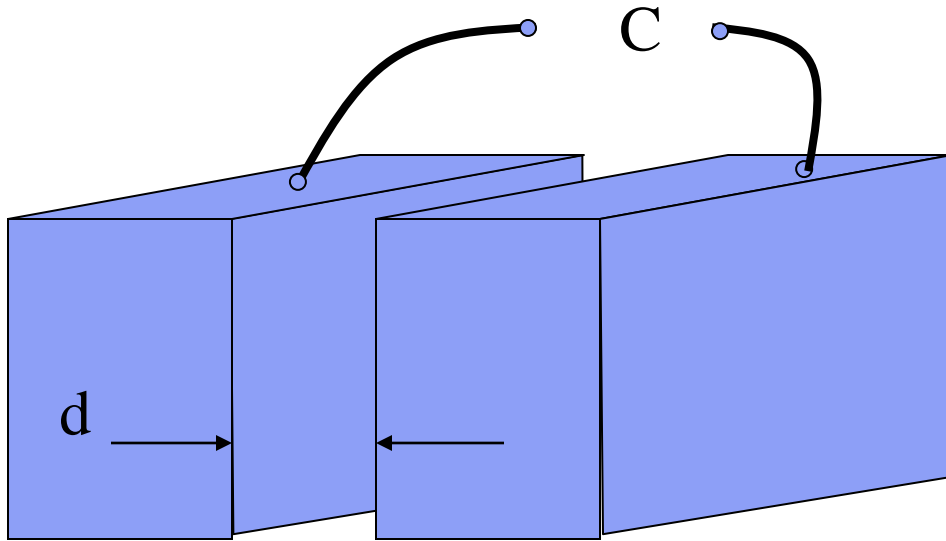
$$C = \frac{\epsilon A}{d}$$

$$C = C_d A$$

where

$$C_d = \frac{\epsilon}{d}$$

Fringe Capacitors

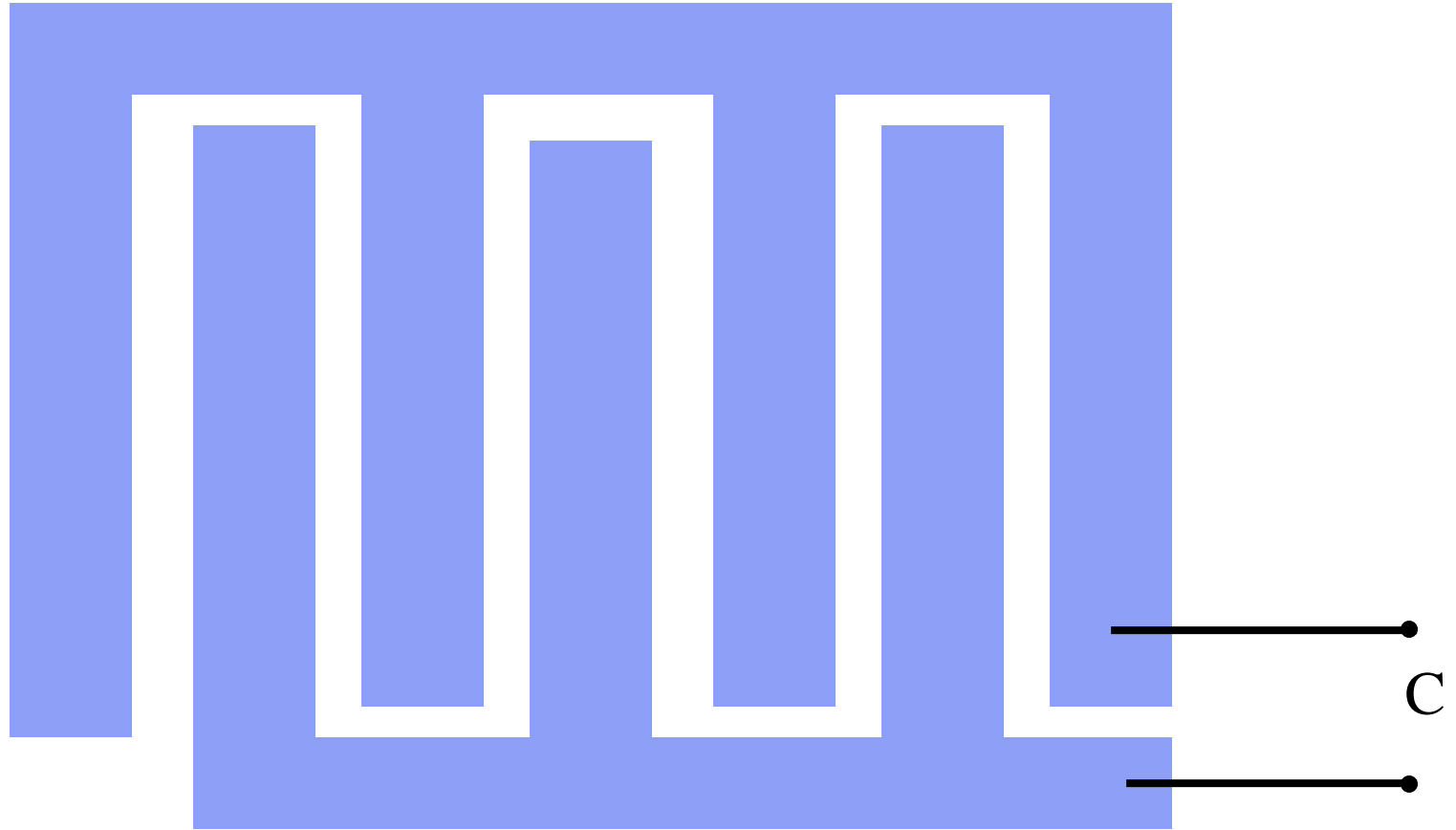


$$C = \frac{\epsilon A}{d}$$

A is the area where the two plates are parallel

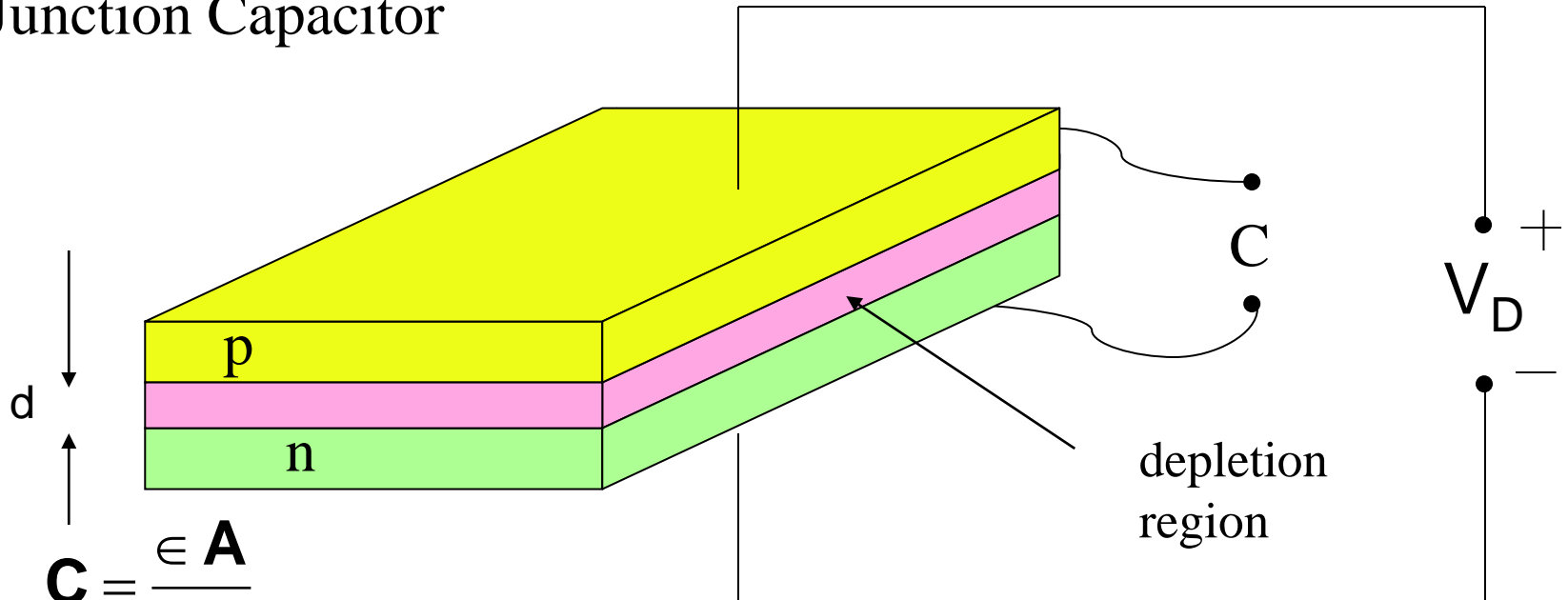
Only a single layer is needed to make fringe capacitors

Fringe Capacitors



Capacitance

Junction Capacitor



$$C = \frac{\epsilon A}{d}$$

ϵ is dielectric constant

$$C = \frac{C_{j0} A}{\left(1 - \frac{V_D}{\phi_B}\right)^n} \quad \text{for } V_{FB} < \frac{\phi_B}{2}$$

- Note: d is voltage dependent
- capacitance is voltage dependent
- usually parasitic caps
- varicaps or varactor diodes exploit voltage dep. of C

C_{j0} is the zero-bias junction capacitance density

Model parameters $\{C_{j0}, n, \phi_B\}$ Design parameters $\{A\}$

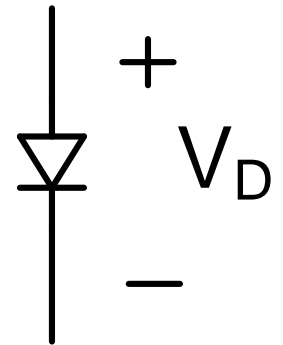
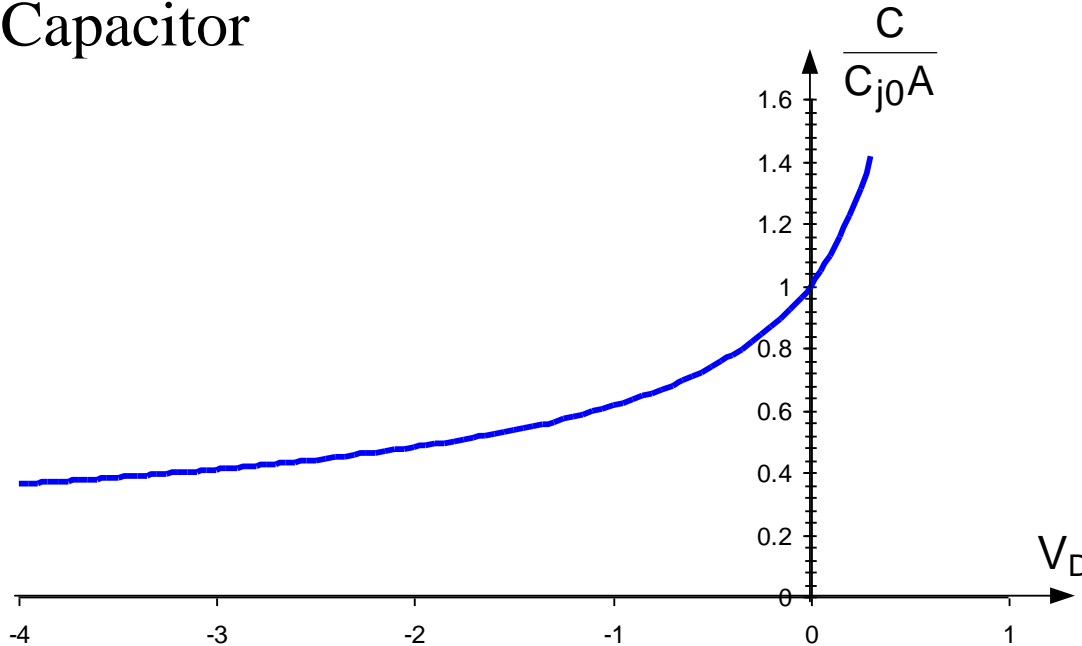
$$\phi_B \cong 0.6V$$

$$n \simeq 0.5$$

$$C_{j0} \text{ highly process dependent around } 500\text{aF}/\mu\text{m}^2$$

Capacitance

Junction Capacitor



$$C = \frac{C_{j0A}}{\left(1 - \frac{V_D}{\Phi_B}\right)^n} \quad \text{for } V_{FB} < \frac{\Phi_B}{2}$$

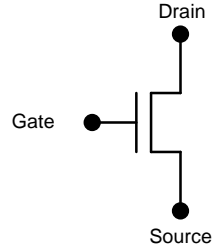
Voltage dependence is substantial

$$\Phi_B \cong 0.6V \quad n \cong 0.5$$

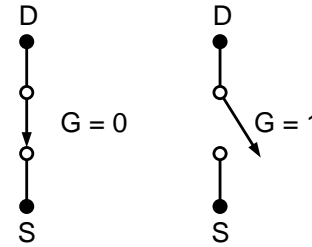
Basic Devices and Device Models

- Resistor
- Diode
- Capacitor
- MOSFET
- BJT

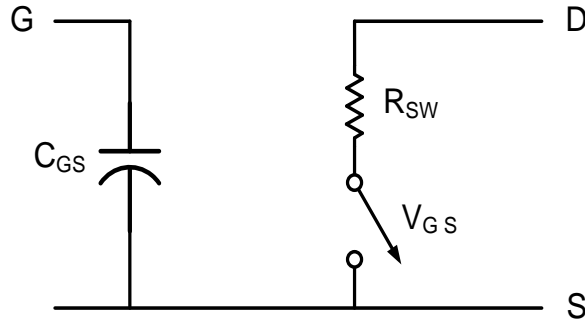
Summary of Existing Models (for n-channel)



1. Switch-Level model

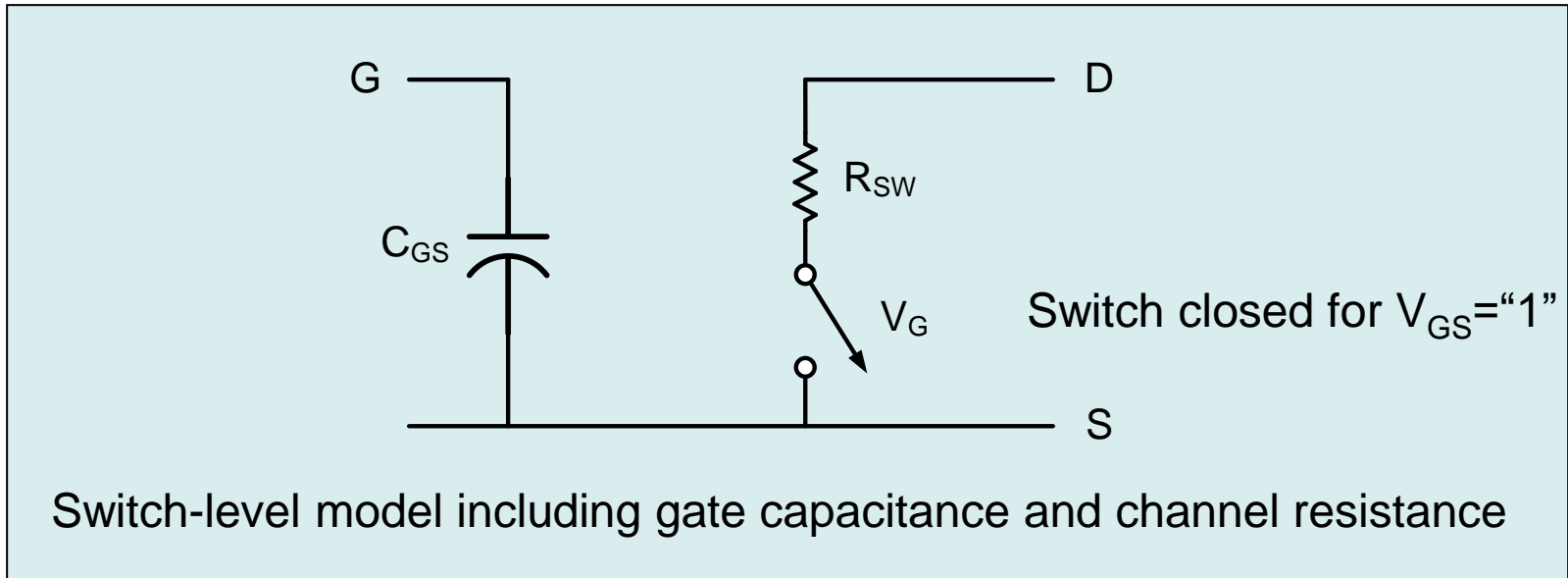


2. Improved switch-level model



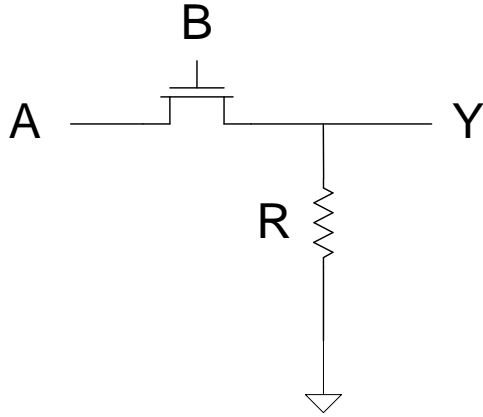
*Switch closed for $|V_{GS}| = \text{large}$
Switch open for $|V_{GS}| = \text{small}$*

Improved Switch-Level Model

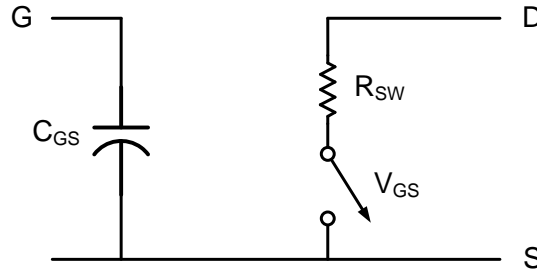


- Connect the gate capacitance to the source to create lumped model
- Still neglect bulk connection

Limitations of Existing MOSFET Models



What is Y when A=B=VDD

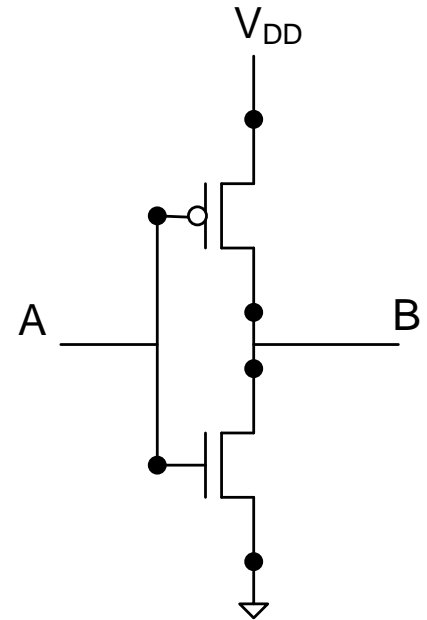


For minimum-sized devices in a 0.5 μ process with $V_{DD}=5V$

$$C_{GS} \cong 1.5fF$$

$$R_{sw} \cong \left. \begin{array}{l} 2K\Omega \text{ n-channel} \\ 6K\Omega \text{ p-channel} \end{array} \right\}$$

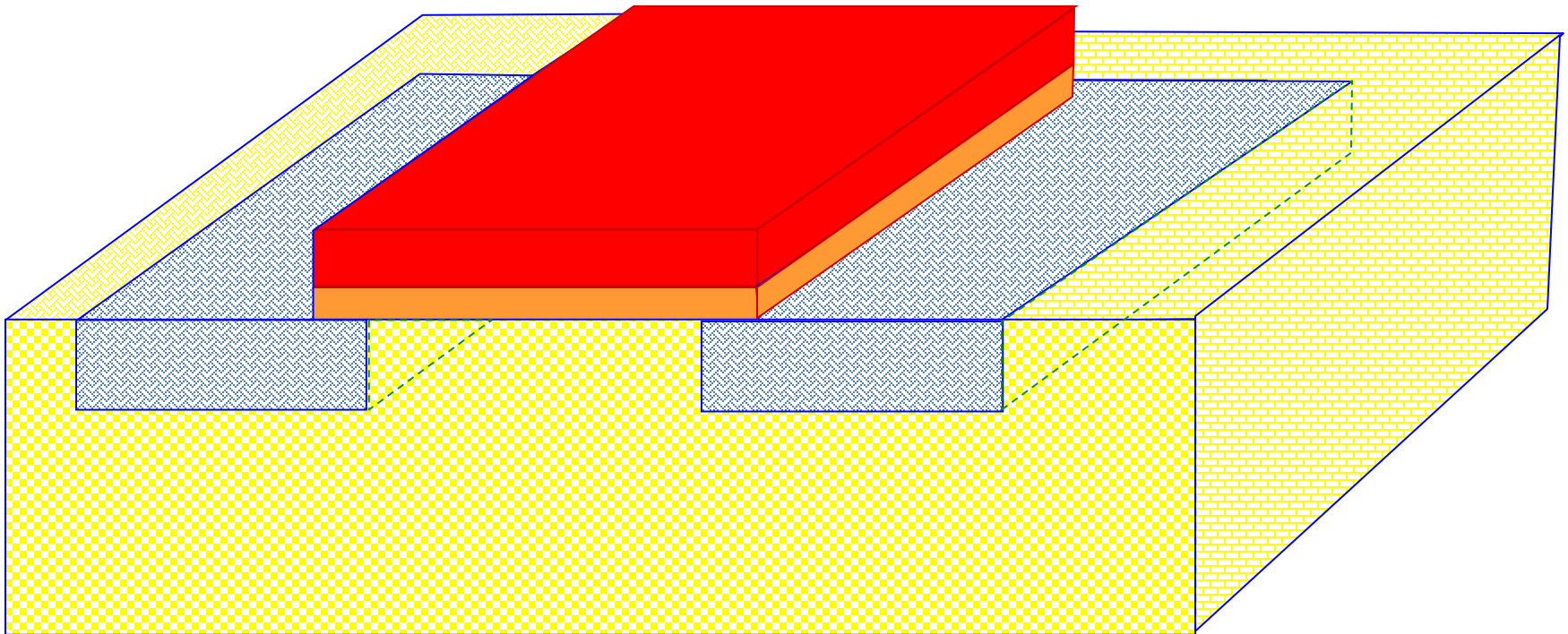
What is R_{sw} if MOSFET is not minimum sized?



What is power dissipation if A is stuck at an intermediate voltage?

Better Model of MOSFET is Needed!

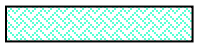
n-Channel MOSFET



Poly



Gate oxide

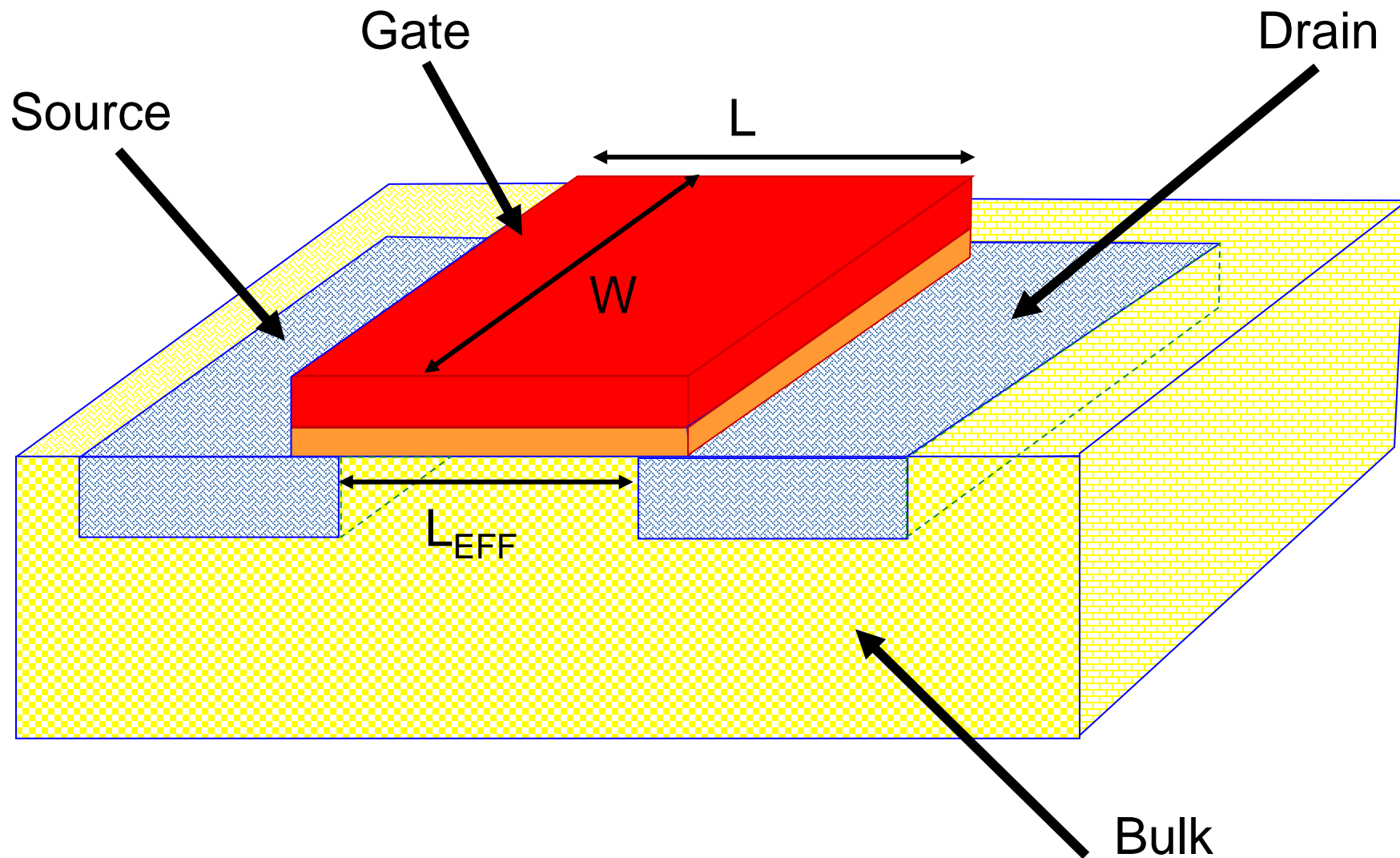


n-active

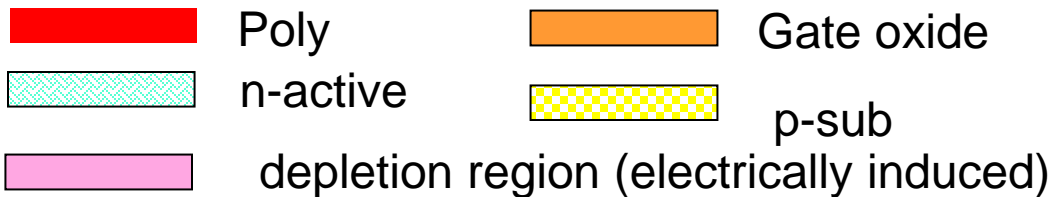
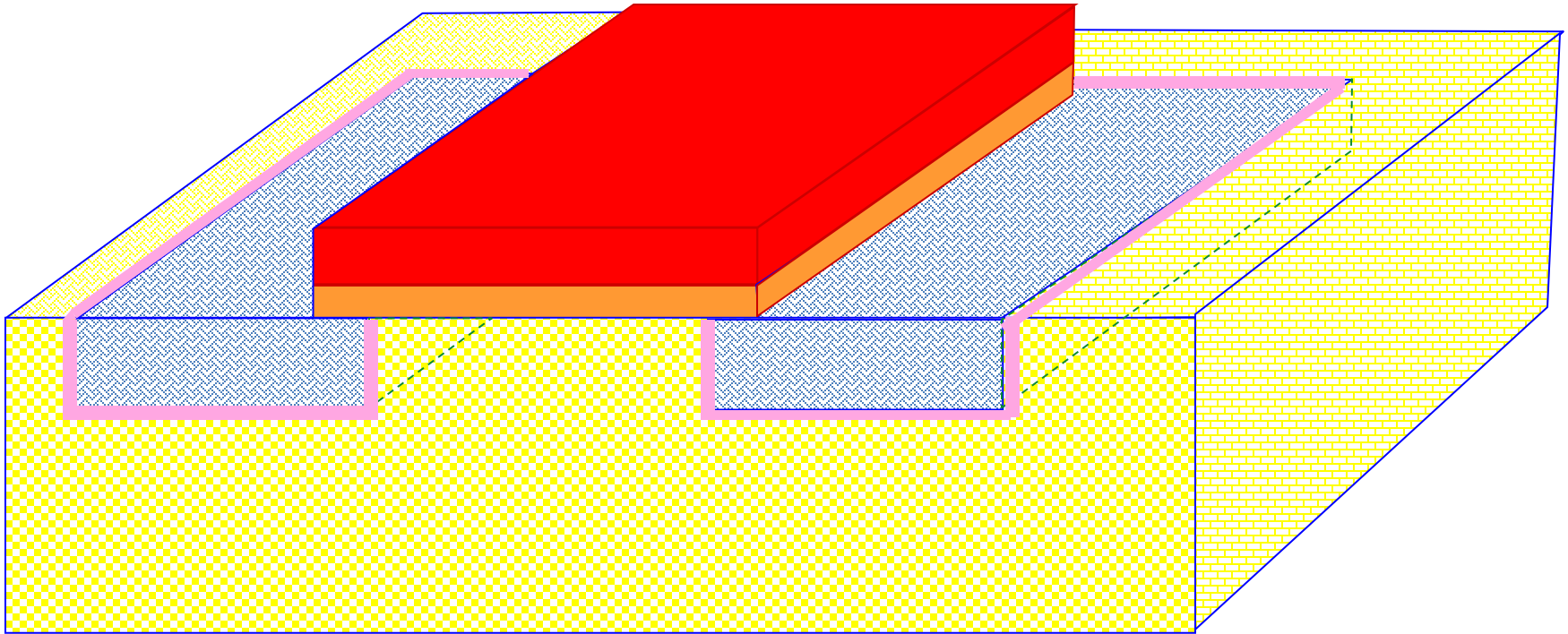


p-sub

n-Channel MOSFET

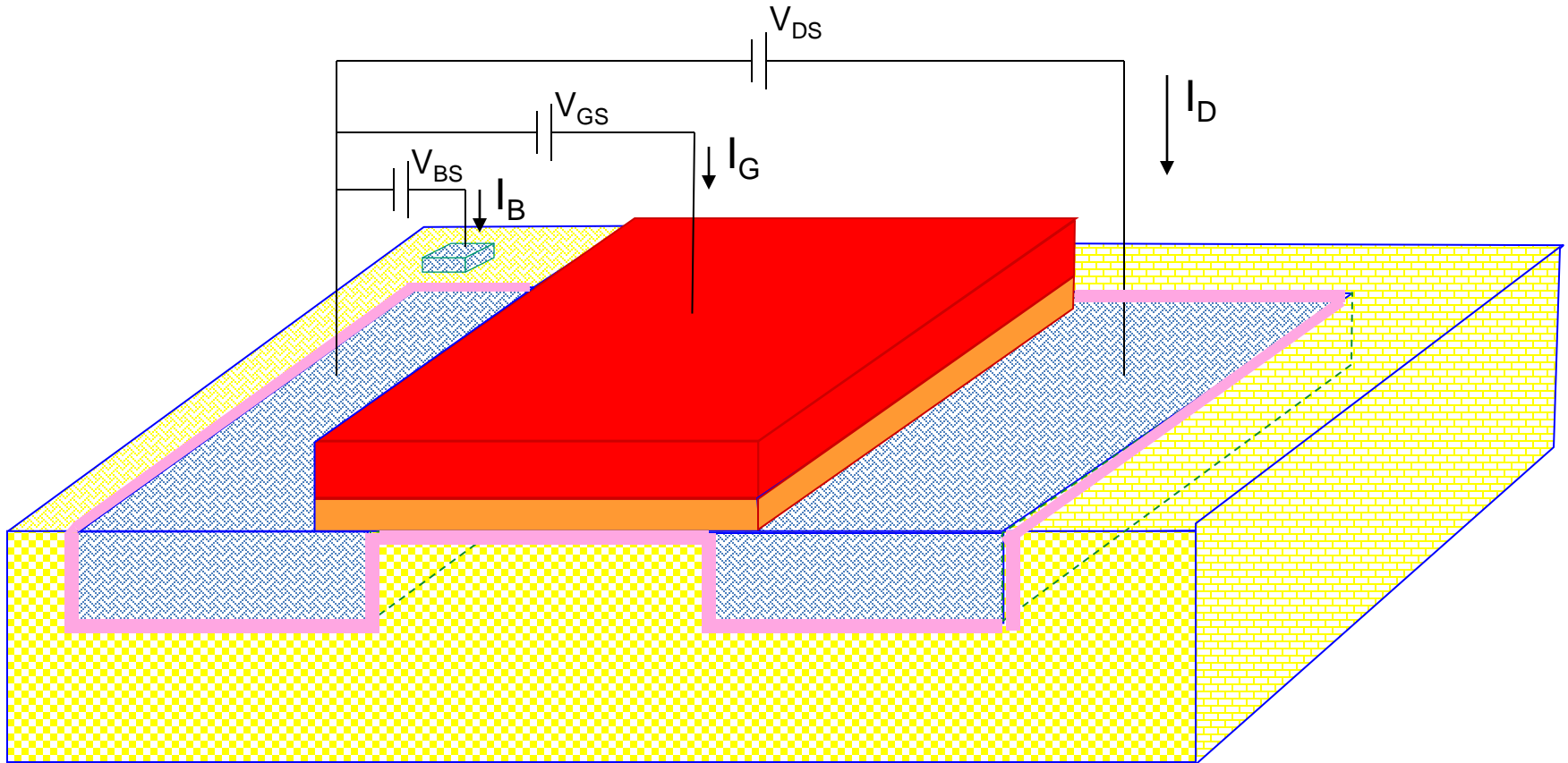


n-Channel MOSFET



- In what follows assume all pn junctions reverse biased (almost always used this way)
- Extremely small reverse bias pn junction current can be neglected in most applications

n-Channel MOSFET Operation and Model

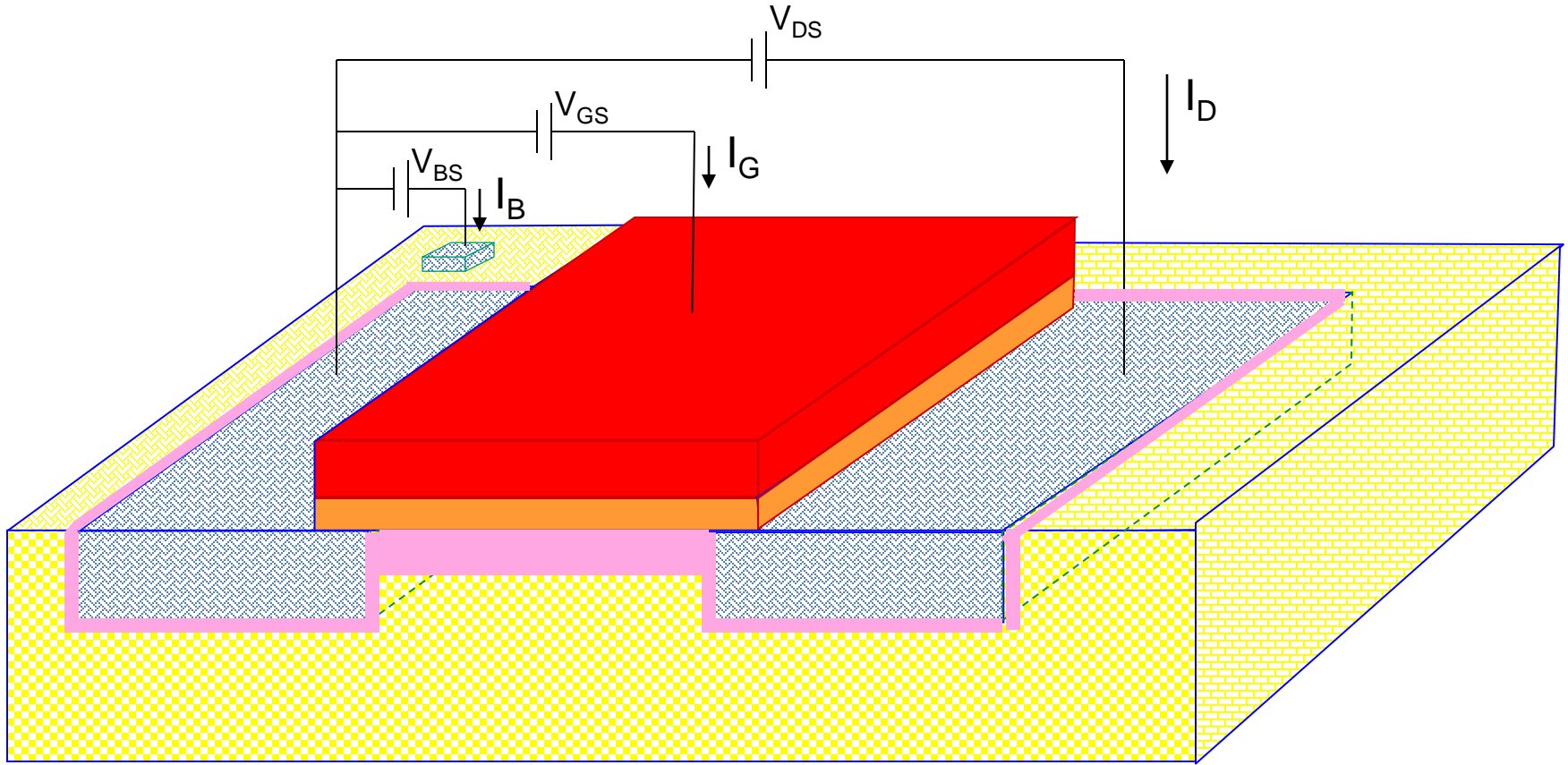


Apply small V_{GS}
(V_{DS} and V_{BS} assumed to be small)

Depletion region electrically induced in channel
Termed "cutoff" region of operation

$I_D=0$
 $I_G=0$
 $I_B=0$

n-Channel MOSFET Operation and Model

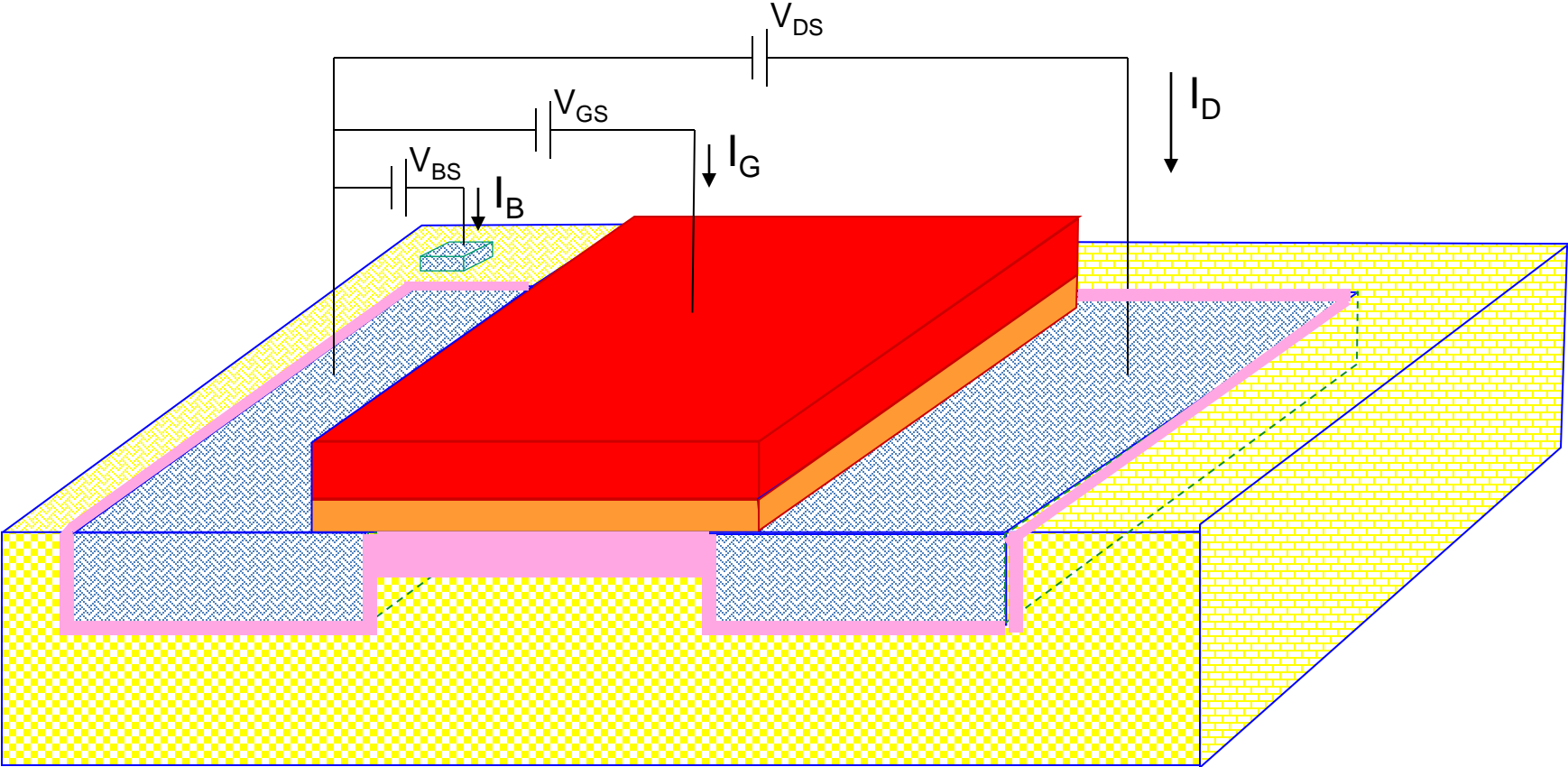


Increase V_{GS}
(V_{DS} and V_{BS} assumed to be small)

Depletion region in channel becomes larger

$I_D=0$
 $I_G=0$
 $I_B=0$

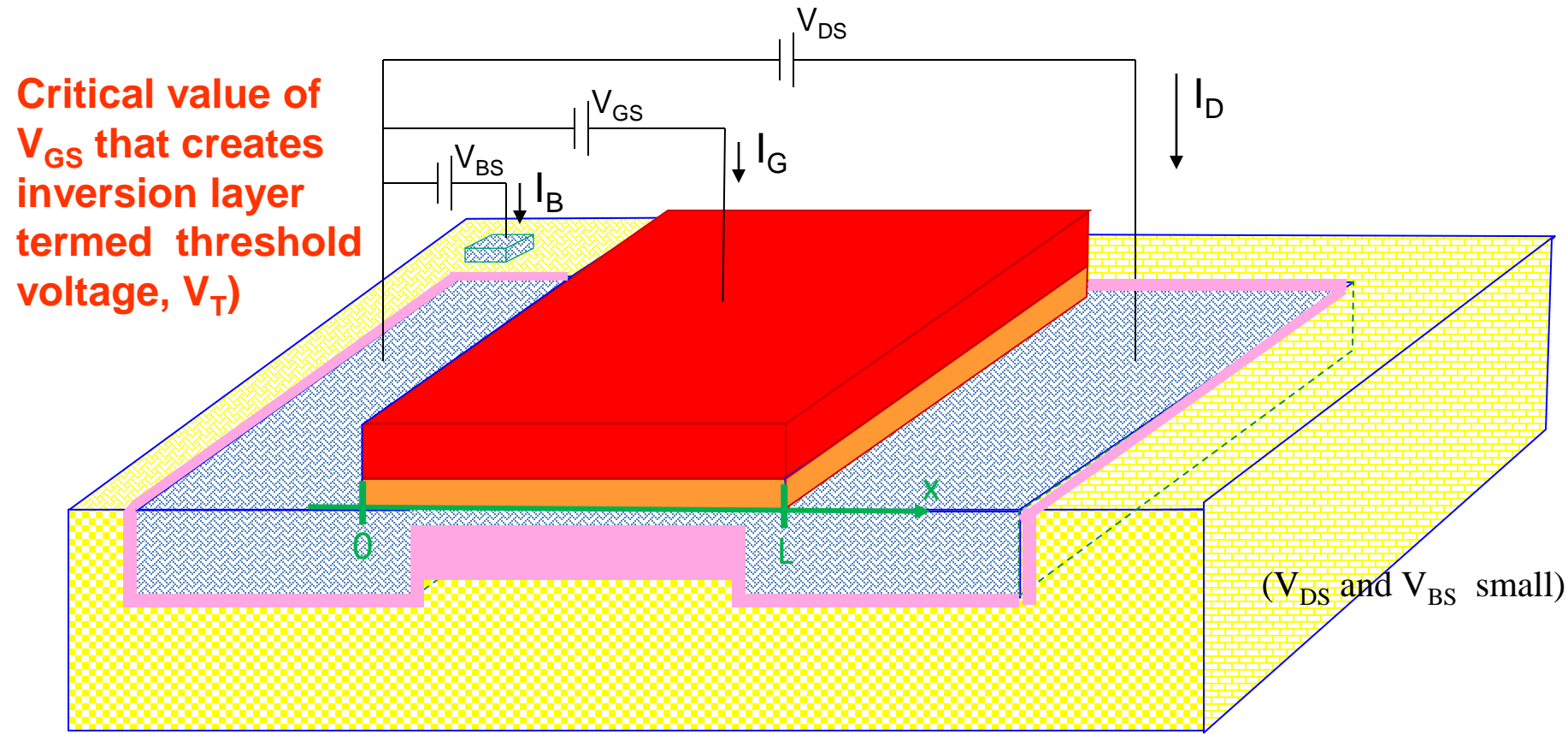
n-Channel MOSFET Operation and Model



$I_D=0$
$I_G=0$
$I_B=0$

Model in Cutoff Region

n-Channel MOSFET Operation and Model



Critical value of V_{GS} that creates inversion layer termed threshold voltage, V_T)

Increase V_{GS} more

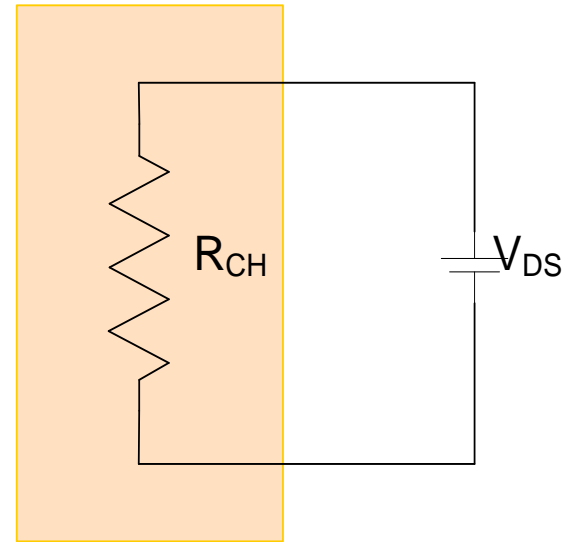
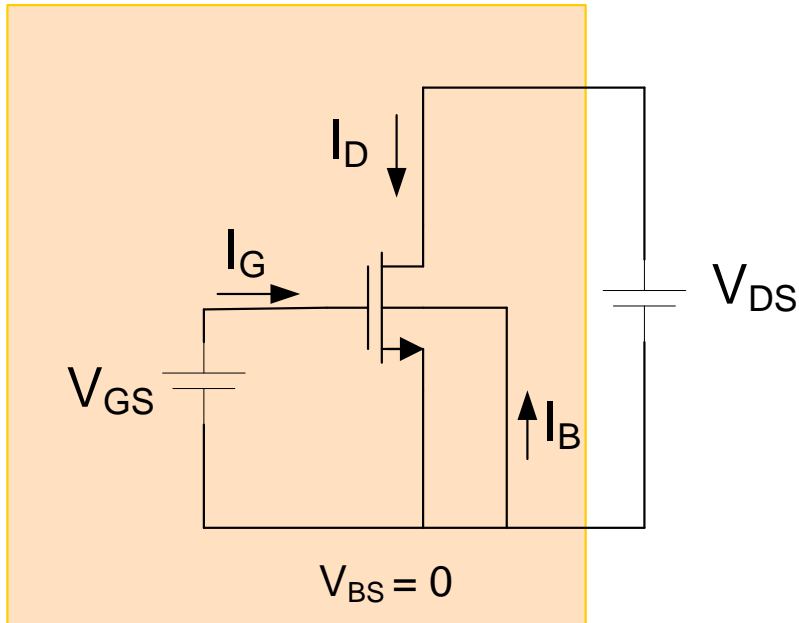
- Inversion layer forms in channel
- Inversion layer will support current flow from D to S
- Channel behaves as thin-film resistor

$$I_D R_{CH} = V_{DS}$$

$$I_G = 0$$

$$I_B = 0$$

Triode Region of Operation



For V_{DS} small

$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$$

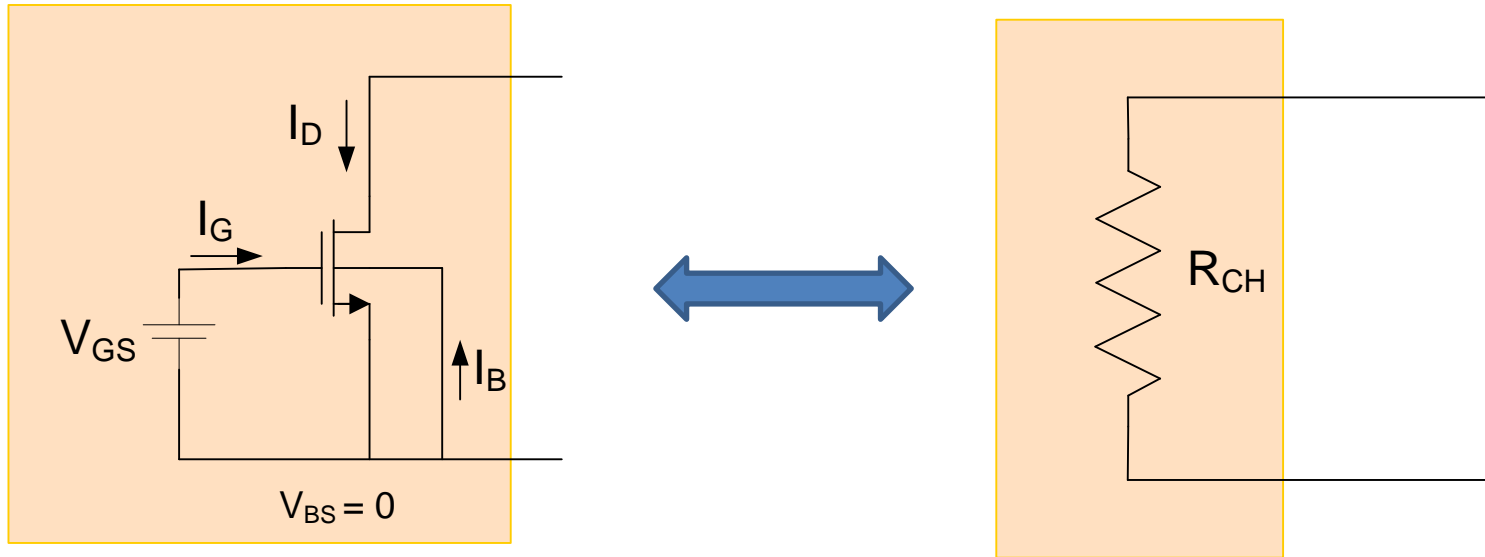
$$I_D = \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS}$$

$$I_G = I_B = 0$$

Behaves as a resistor between drain and source

Model in Deep Triode Region

Triode Region of Operation

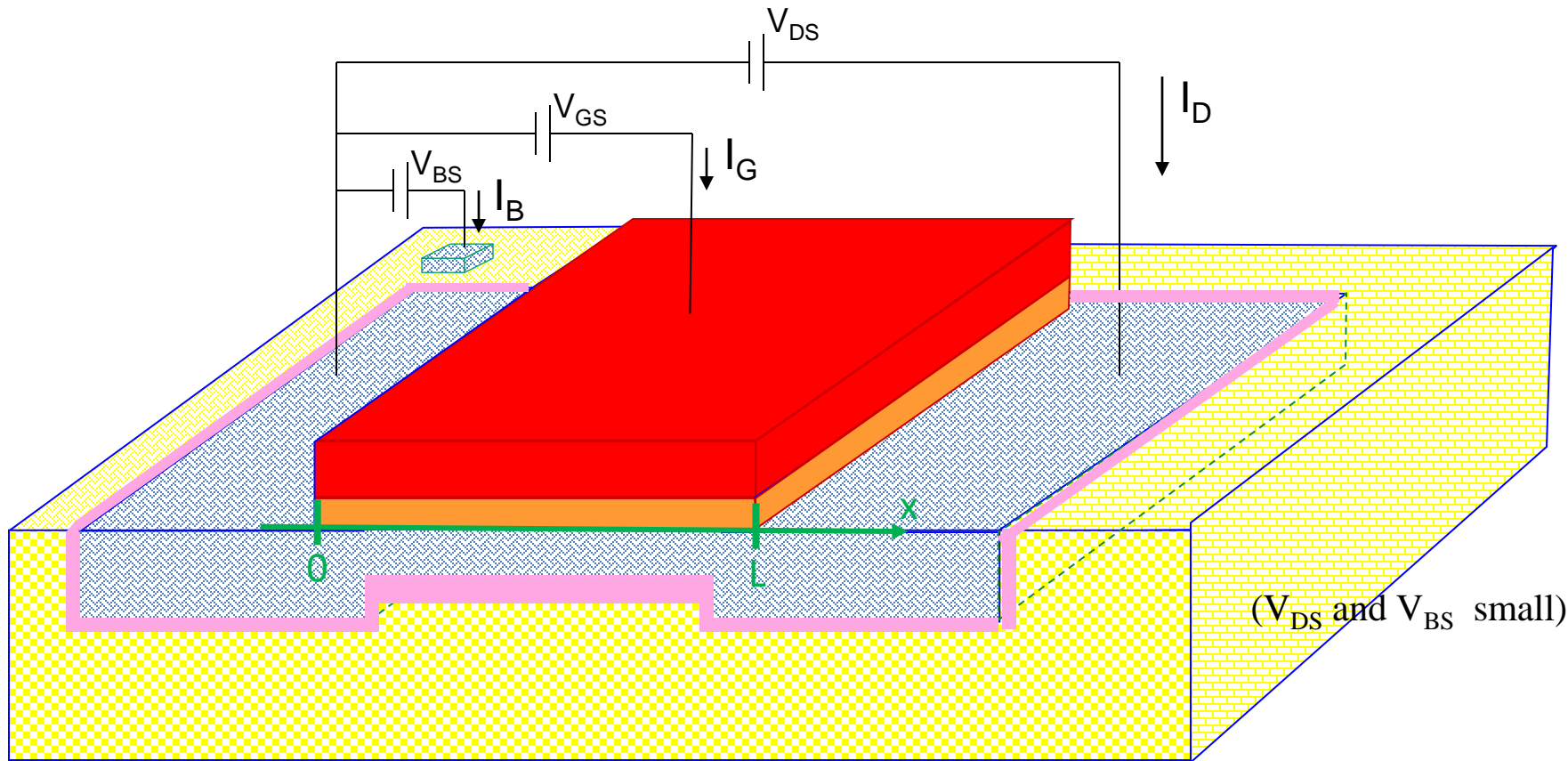


For V_{DS} small

$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$$

Resistor is controlled by the voltage V_{GS}
Termed a "Voltage Controlled Resistor" (VCR)

n-Channel MOSFET Operation and Model



$V_{GC}(x)$ approx. constant for small V_{DS}

Increase V_{GS} more

Inversion layer in channel thickens

R_{CH} will decrease

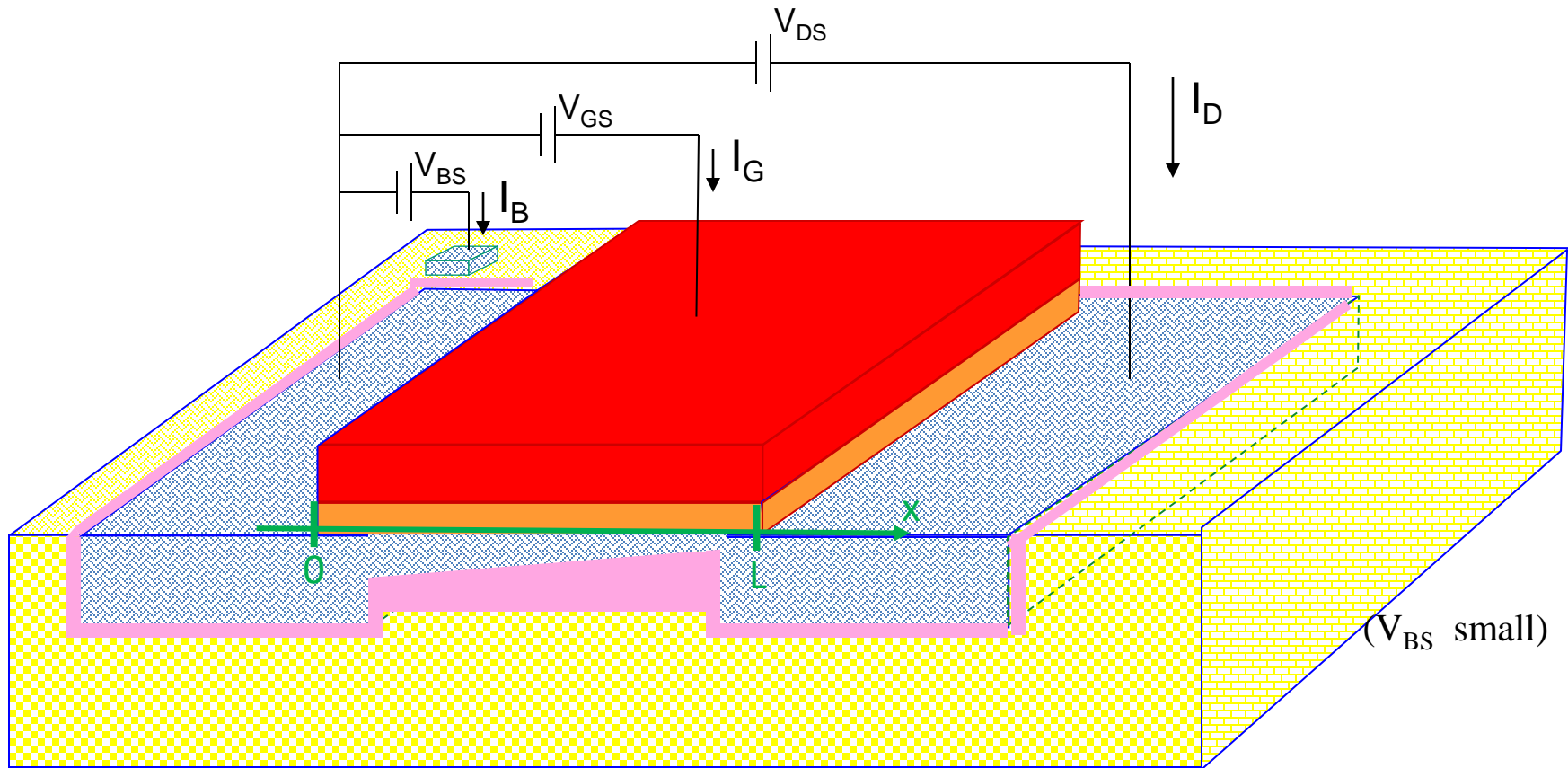
Termed “ohmic” or “triode” region of operation

$$I_D R_{CH} = V_{DS}$$

$$I_G = 0$$

$$I_B = 0$$

n-Channel MOSFET Operation and Model



Increase V_{DS}

$V_{GC}(x)$ changes with x for larger V_{DS}

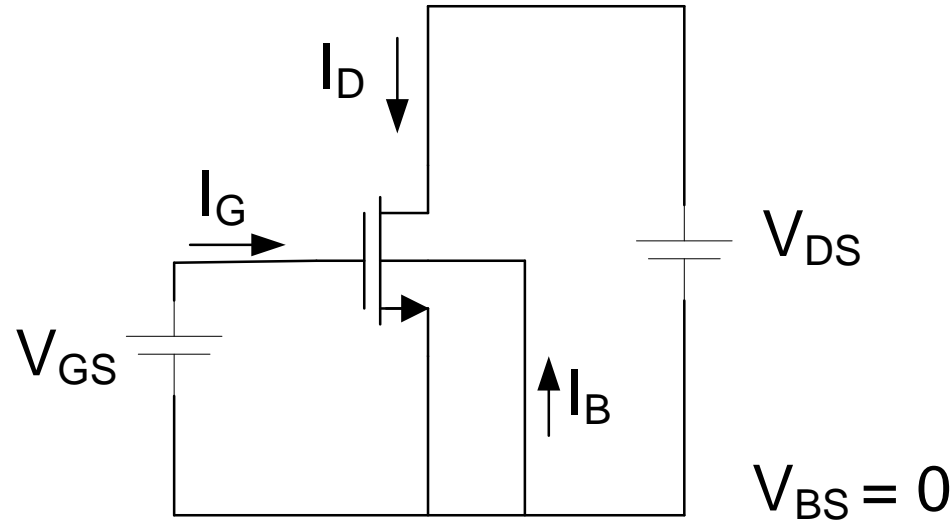
Inversion layer thins near drain
 I_D no longer linearly dependent upon V_{DS}
 Still termed “ohmic” or “triode” region of operation

$$I_D = ?$$

$$I_G = 0$$

$$I_B = 0$$

Triode Region of Operation



For V_{DS} larger

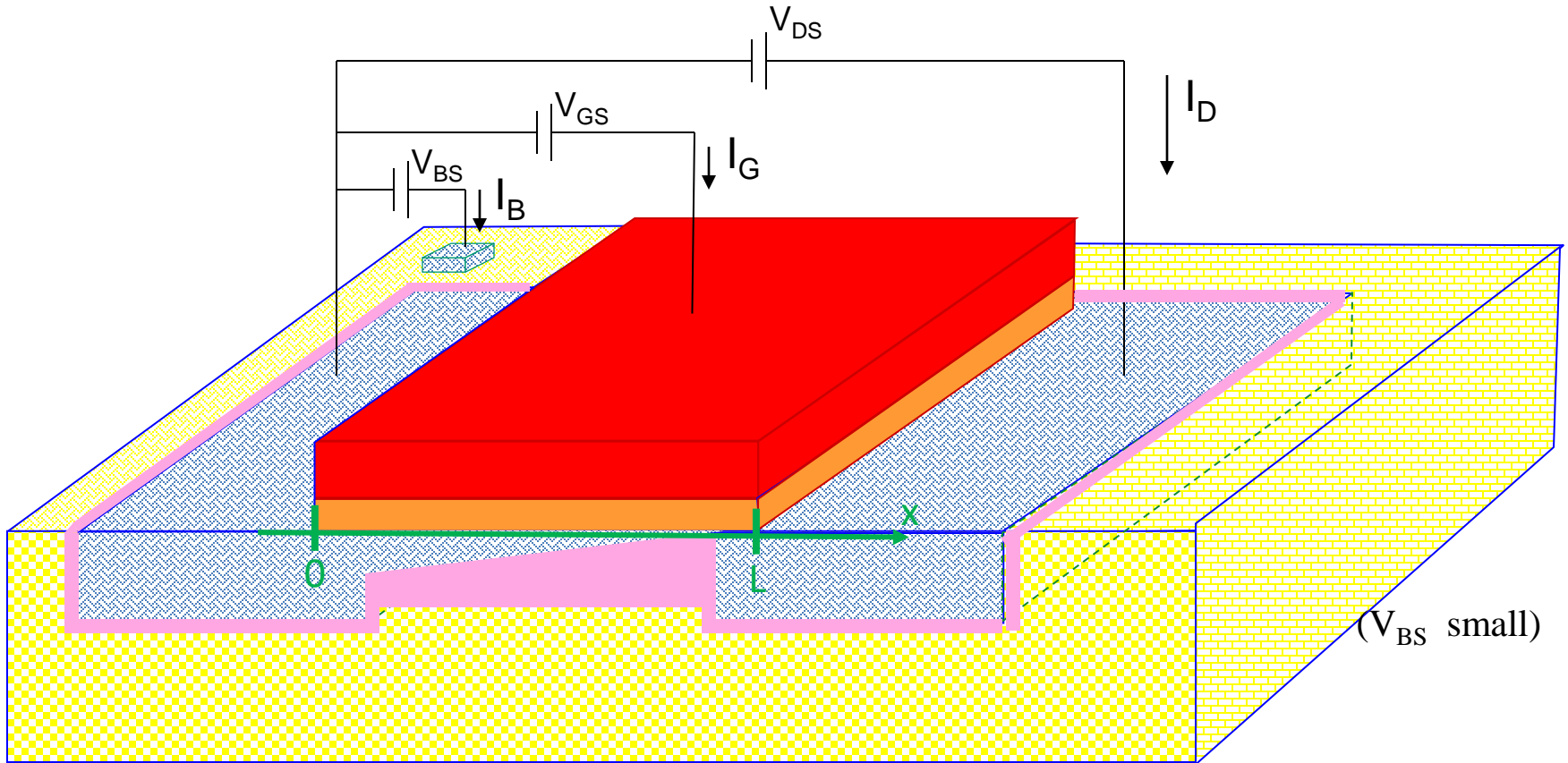
~~$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$$~~

$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

$$I_G = I_B = 0$$

Model in Triode Region

n-Channel MOSFET Operation and Model



Increase V_{DS} even more

$V_{GC}(L) = V_{TH}$ when channel saturates

Inversion layer disappears near drain

Termed "saturation" region of operation

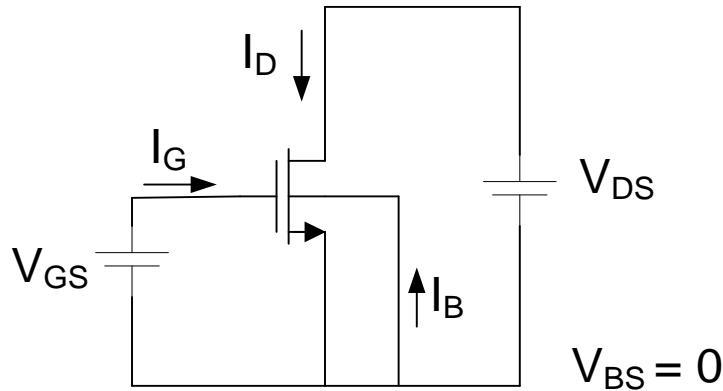
Saturation first occurs when $V_{DS} = V_{GS} - V_{TH}$

$I_D = ?$

$I_G = 0$

$I_B = 0$

Saturation Region of Operation



$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

or equivalently

$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{GS} - V_{TH}}{2} \right) (V_{GS} - V_{TH})$$

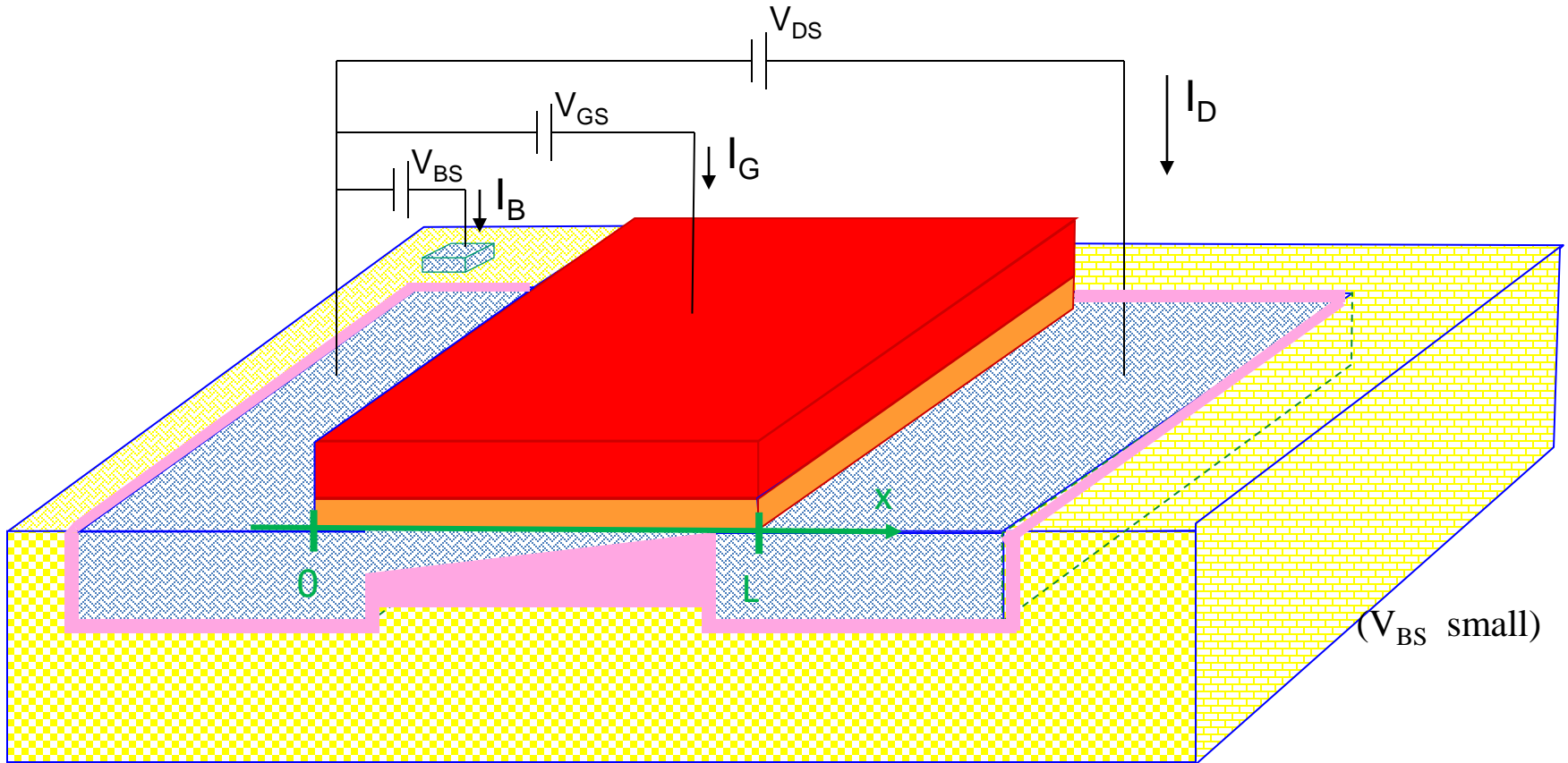
or equivalently

$$I_D = \frac{\mu C_{OX} W}{2L} (V_{GS} - V_{TH})^2$$

$$I_G = I_B = 0$$

For V_{DS} at onset of saturation

n-Channel MOSFET Operation and Model



Increase V_{DS} even more (beyond $V_{GS} - V_{TH}$)

Nothing much changes !!

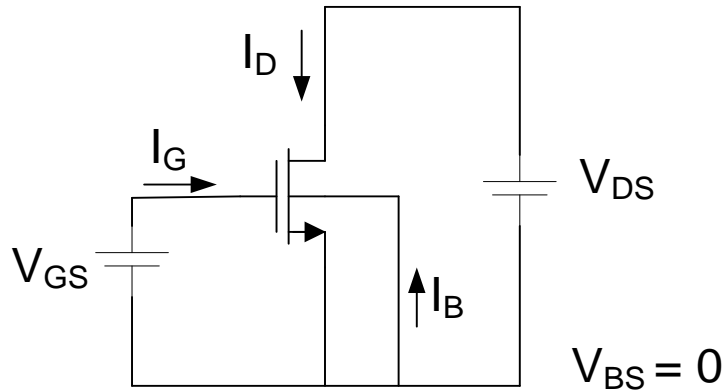
Termed "saturation" region of operation

$$I_D = ?$$

$$I_G = 0$$

$$I_B = 0$$

Saturation Region of Operation



For V_{DS} in Saturation

$$I_D = \frac{\mu C_{OX} W}{2L} (V_{GS} - V_{TH})^2$$

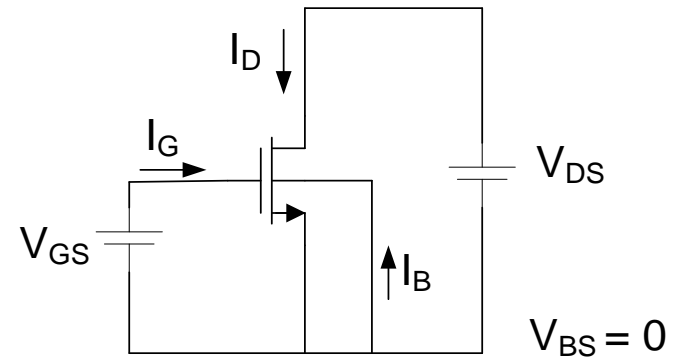
$$I_G = I_B = 0$$

Model in Saturation Region

Model Summary

n-channel MOSFET

Notation change: $V_T = V_{TH}$, don't confuse V_T with $V_t = kT/q$



$$I_D = \begin{cases} 0 & V_{GS} \leq V_T & \text{Cutoff} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T & \text{Triode} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T & \text{Saturation} \end{cases}$$

$$I_G = I_B = 0$$

Model Parameters: $\{\mu, V_T, C_{OX}\}$ Design Parameters : $\{W, L\}$

This is a piecewise model (not piecewise linear though)

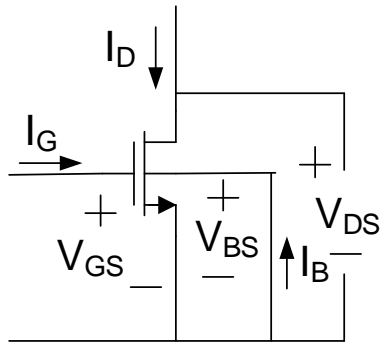
Piecewise model is continuous at transition between regions

(Deep triode special case of triode where V_{DS} is small $R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_T) \mu C_{OX}}$)

Note: This is the third model we have introduced for the MOSFET

Model Summary

n-channel MOSFET



$V_{BS} = 0$

$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{ox} \frac{W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

$$I_G = I_B = 0$$

Observations about this model (developed for $V_{BS}=0$):

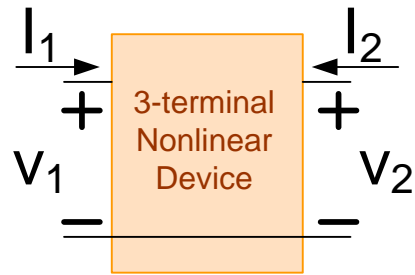
$$I_D = f_1(V_{GS}, V_{DS})$$

$$I_G = f_2(V_{GS}, V_{DS})$$

$$I_B = f_3(V_{GS}, V_{DS})$$

This is a nonlinear model characterized by the functions f_1 , f_2 , and f_3 where we have assumed that the port voltages V_{GS} and V_{DS} are the independent variables and the drain currents are the dependent variables

General Nonlinear Models

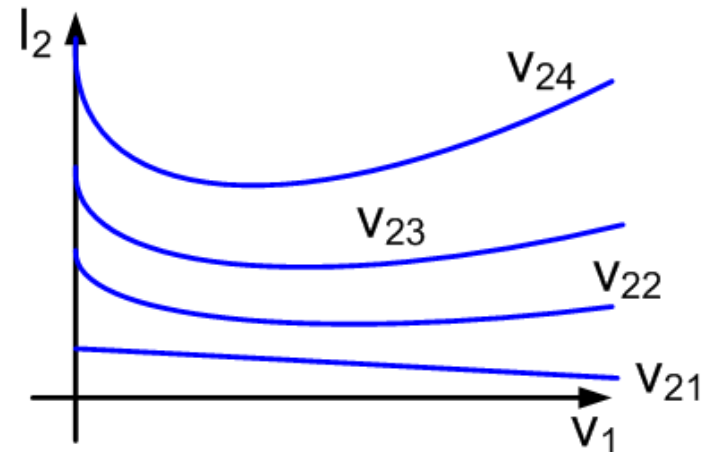
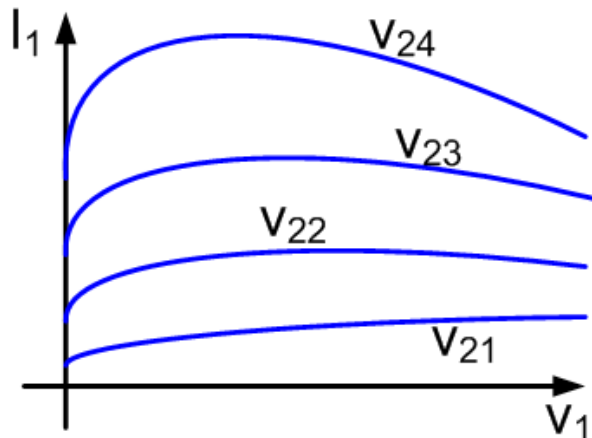


$$I_1 = f_1(V_1, V_2)$$

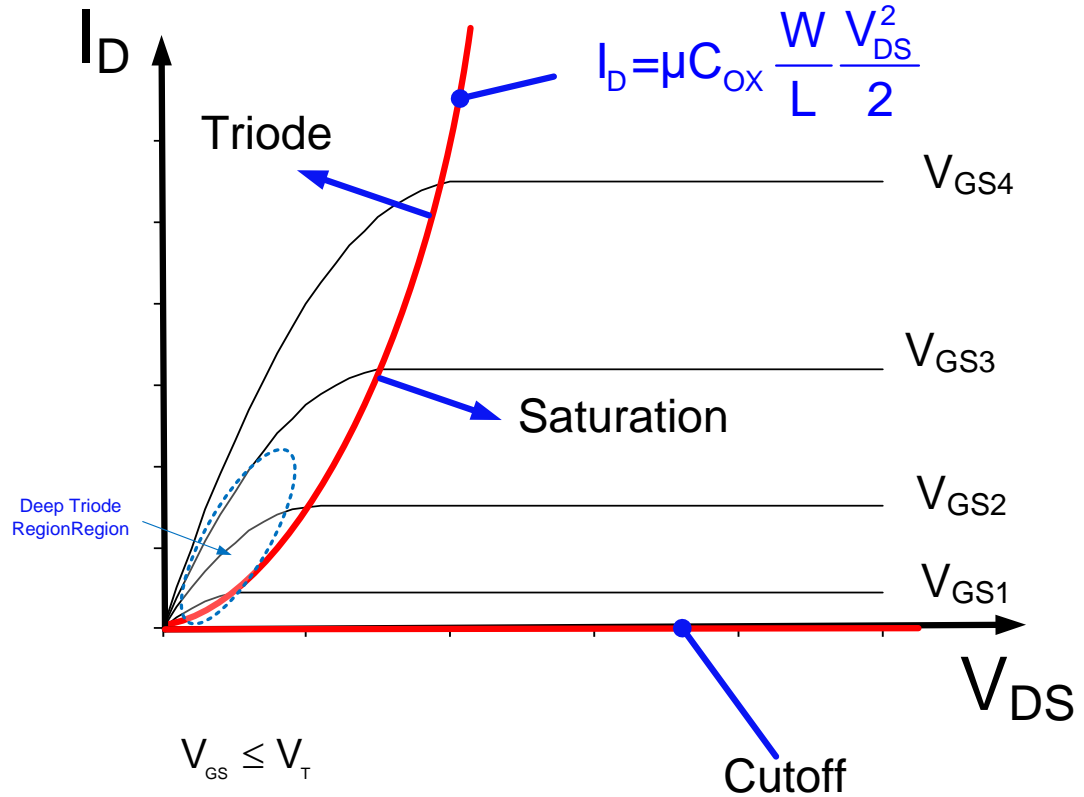
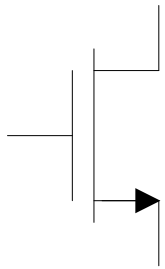
$$I_2 = f_2(V_1, V_2)$$

I_1 and I_2 are 3-dimensional relationships which are often difficult to visualize

Two-dimensional representation of 3-dimensional relationships



Graphical Representation of MOS Model



$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{ox} \frac{W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\ \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T \end{cases}$$

$$I_G = I_B = 0$$

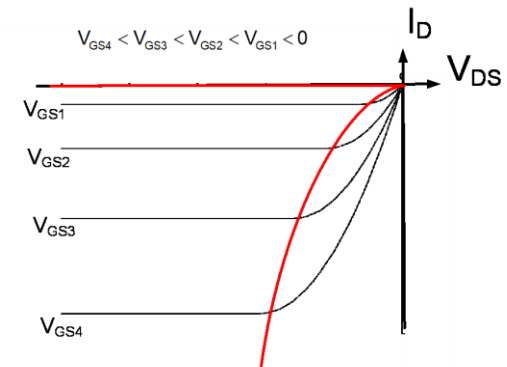
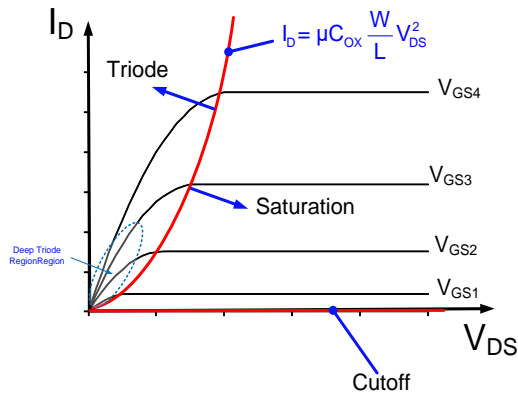
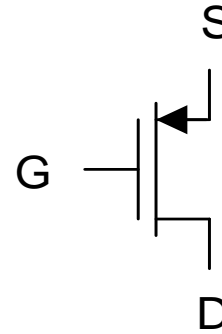
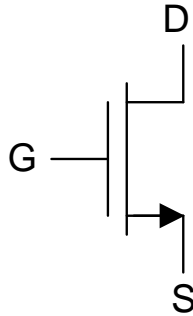
Parabola separated triode and saturation regions and corresponds to $V_{DS} = V_{GS} - V_T$



Stay Safe and Stay Healthy !

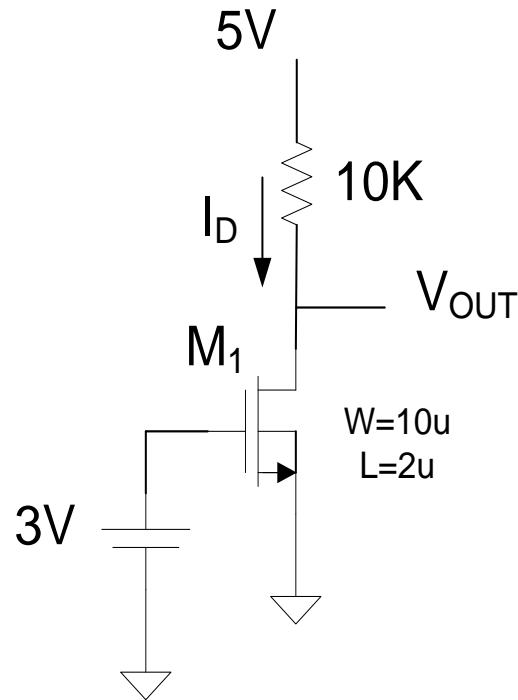
End of Lecture 16

PMOS and NMOS Models



- Functional form identical, sign changes and parameter values different
- Will give details about p-channel model later

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

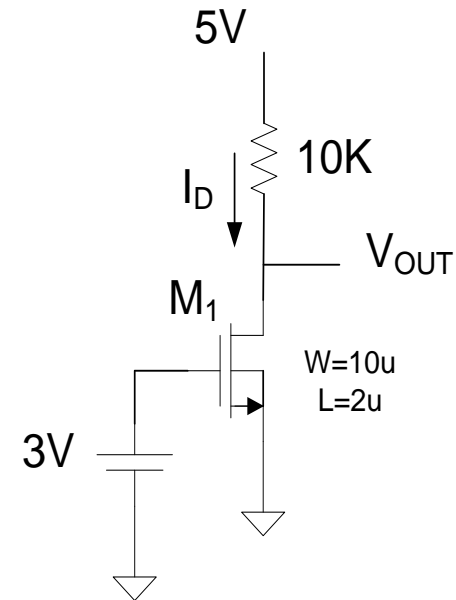


Solution:

Since $V_{GS} > V_T$, M_1 is operating in either saturation or triode region

Strategy will be to guess region of operation, solve, and then verify region

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T=1V$ and $\mu C_{OX}=100\mu AV^{-2}$



Solution:

Guess M_1 in saturation

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{2L} (3 - V_T)^2$$

Required verification: $V_{DS} > V_{GS} - V_T$

Can eliminate I_D between these 2 equations to obtain V_{OUT}

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

Guess M_1 in saturation

Required verification: $V_{DS} > V_{GS} - V_T$

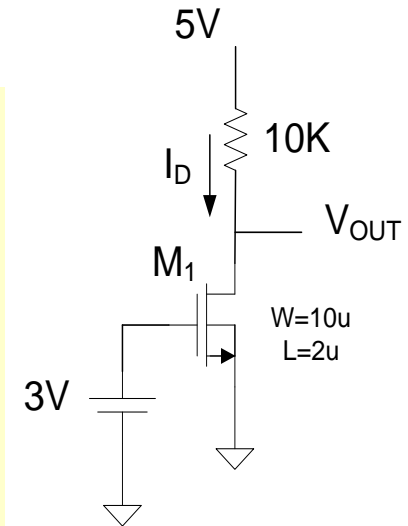
$$\left. \begin{aligned} 5V &= I_D 10K + V_{OUT} \\ I_D &= \frac{\mu C_{OX} W}{2L} (3 - V_T)^2 \end{aligned} \right\}$$

$$V_{OUT} = 5V - 10K \left[\frac{100\mu AV^{-2} 10\mu}{2 \cdot 2\mu} (2V)^2 \right]$$

$$V_{OUT} = -5V$$

Verification: $V_{DS} = V_{OUT}$

$-5 >? 2V - - 0$ No! So verification fails and Guess of region is invalid



Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

Guess M_1 in triode

Required verification: $V_{DS} < V_{GS} - V_T$

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{L} \left(3 - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

$$V_{OUT} = 5V - 10K \left[\frac{100\mu AV^{-2} 10\mu}{2\mu} \left(2V - \frac{V_{OUT}}{2} \right) V_{OUT} \right]$$

$$V_{OUT} = 5V - \left[5 \left(2V - \frac{V_{OUT}}{2} \right) V_{OUT} \right]$$

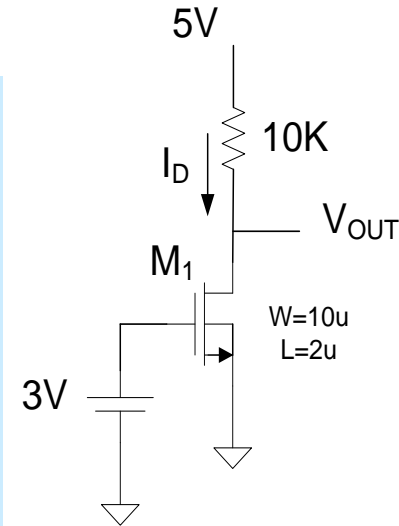
Solving for V_{OUT} , obtain

$$V_{OUT} = 0.515V$$

Verification: $V_{DS} = V_{OUT}$

$0.515 < 2V$ Yes!

So verification succeeds and triode region is valid



$$V_{OUT} = 0.515V$$